Title:	RPAD- Resistive Attenuator Design
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Abstract:	This brief memo provides technical back-up to the RPAD computer program.
Date:	16 July 2011
Version:	1.0

1 Introduction

Impedance-matched resistive attenuator design is fundamentally based on the theory of image parameter filters as discussed in §6.2 of [1]. When properly designed, each port of the attenuator provides an ideal impedance match at each respective port.

Given the desired input and output port impedances represented by R_{in} and R_{out} respectively, the minimum attenuator gain that can be realized under the matched condition is given by

$$A_{\min} = \frac{20}{2.3026}\cosh^{-1}\left(\sqrt{\frac{R_{in}}{R_{out}}}\right) \quad \text{dB} \tag{1}$$

where it has been assumed that $R_{in} \geq R_{out}$.

The design equations for the tee-attenuator shown in Figure 1 are as follows:

$$\begin{aligned} R_2 &= \frac{\sqrt{R_{in}R_{out}}}{\sinh\left(A_{np}\right)} \\ R_1 &= R_2\left[\sqrt{\frac{R_{in}}{R_{out}}}\cosh\left(A_{np}\right)-1\right] \\ R_3 &= R_2\left[\sqrt{\frac{R_{out}}{R_{in}}}\cosh\left(A_{np}\right)-1\right] \end{aligned} \tag{2}$$

and A_{np} is the desired power attenuation factor in units of nepers. Given the desired attenuation A_{dB} in dB,

$$A_{np} = \frac{2.3026}{20}A_{dB} \tag{3}$$

The design equations for the π-attenuator shown in Figure 2 are similarly given by

$$\begin{aligned} R_2 &= \sqrt{R_{in}R_{out}}\sinh\left(A_{np}\right) \\ R_1 &= R_2\left[\sqrt{\frac{R_{out}}{R_{in}}}\cosh\left(A_{np}\right)-1\right]^{-1} \\ R_3 &= R_2\left[\sqrt{\frac{R_{in}}{R_{out}}}\cosh\left(A_{np}\right)-1\right]^{-1} \end{aligned} \tag{4}$$

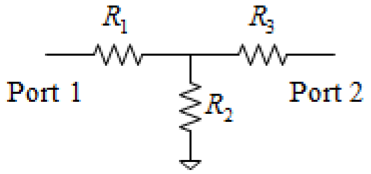


Figure 1 Tee resistive attenuator

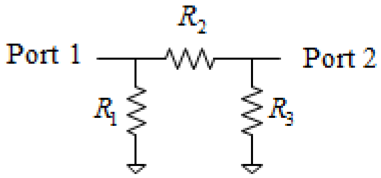


Figure 2 π resistive attenuator

2 Resultant Characteristics

Since resistors are normally only available in discrete values, it is helpful to compute the resultant attenuator performance once these real-world values have been selected. This can be done in a number of ways by using *z*-, *y*-, or *ABCD*- parameters. The method adopted here is the latter.

The *ABCD* matrices for the two attenuator types are given by [2]

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\pi} = \begin{bmatrix} 1 & 0 \\ R_1^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ R_3^{-1} & 1 \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{R_2}{R_3}\right) & \left(R_2\right) \\ \left(R_1^{-1} + R_3^{-1} + \frac{R_2}{R_1 R_3}\right) & \left(1 + \frac{R_2}{R_1}\right) \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{tee} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ R_2^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{R_1}{R_2}\right) & \left(R_1 + R_3 + \frac{R_1 R_3}{R_2}\right) \\ R_2^{-1} & \left(1 + \frac{R_3}{R_2}\right) \end{bmatrix} \tag{6}$$

Given these results, it is a simple matter to compute the corresponding S-parameters from the following relationships:

$$S_{11} = \frac{AR_{out} + B - CR_{in}R_{out} - DR_{in}}{d} \tag{7}$$

$$S_{12} = \frac{2\left(AD - BC\right)\sqrt{R_{in}R_{out}}}{d} \tag{8}$$

$$S_{21} = \frac{2\sqrt{R_{in}R_{out}}}{d} \tag{9}$$

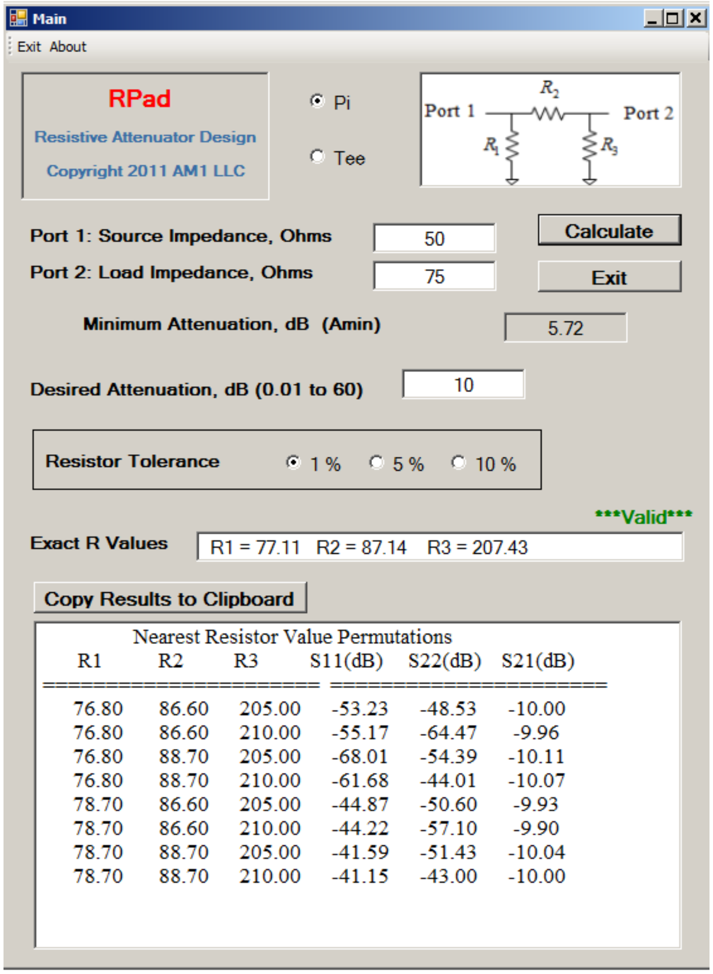
$$S_{22} = \frac{-AR_{out} + B - CR_{in}R_{out} + DR_{in}}{d} \tag{10}$$

where

$$d = AR_{out} + B + CR_{in}R_{out} + DR_{in} \tag{11}$$

3 Design Example Using RPAD

Assume that a 10 dB Ω-attenuator pad is desired between a source impedance of 50Ω and a load impedance of 75Ω. The parameters are entered into the program form as shown in Figure 3 followed by clicking on the *Calculate* button. As also shown, the minimum possible attenuation possible between these two port impedances is 5.72 dB. The user has a choice to select between 1%, 5%, or 10% standard resistor values and all of the nearest-neighbor resistor values permutated as shown. The user is free to select which attenuator design meets their requirements better.



● Pi
○ Tee

Port 1: Source Impedance, Ohms: 50
Port 2: Load Impedance, Ohms: 75
Minimum Attenuation, dB (Amin): 5.72
Desired Attenuation, dB (0.01 to 60): 10
Resistor Tolerance: ● 1 % ○ 5 % ○ 10 %

Valid

Exact R Values: R1 = 77.11 R2 = 87.14 R3 = 207.43

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          Nearest Resistor Value Permutations
   R1      R2      R3       S11(dB)   S22(dB)   S21(dB)
=======================  =======================
  76.80   86.60   205.00   -53.23   -48.53   -10.00
  76.80   86.60   210.00   -55.17   -64.47    -9.96
  76.80   88.70   205.00   -68.01   -54.39   -10.11
  76.80   88.70   210.00   -61.68   -44.01   -10.07
  78.70   86.60   205.00   -44.87   -50.60    -9.93
  78.70   86.60   210.00   -44.22   -57.10    -9.90
  78.70   88.70   205.00   -41.59   -51.43   -10.04
  78.70   88.70   210.00   -41.15   -43.00   -10.00
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Figure 3 Design example using RPAD

4 References

1. Giacoletto, L.J., *Electronics Designers' Handbook*, 2nd ed., McGraw-Hill Book, 1977.
2. Davis, W. Alan, *Microwave Semiconductor Circuit Design*, Van Nostrand Reinhold, 1984.