The Phase/Frequency Detector

An analysis of phase-locked loop design employing phase frequency detectors.

By James Crawford

The controversial subject of "Divider Time Delay"[1] in recent *RF Design* issues has prompted the following discussion which was presented at a Hughes Aircraft Co. in-plant class on phase-locked loop design. The following analysis is an endorsement of Dr. Egan's explanation[2] where he states that the appearance of the delay-like term is due to the sampling process which is taking place in the phase-locked loop, not the transport delay or any other delay through the divider.

The delay-like terms which was mentioned above is shown in equations (3) and (4) of reference [3]. In this reference it is suggested that "the discrepancy between theory and experiment (in phase-locked loop design) was found to be attributed to divider delay which caused a decrease in phase margin significant enough in many cases to cause unstable loop performance." A decrease in loop phase margin does indeed occur in these phase-locked loops but the sole mechanism is a result of the sampling process which is taking place in the closed loop.

A rigorous analysis of phase-locked loop design employing phase-frequency detectors necessitates a detailed examination of the operation fundamentals. Rather than deal immediately with the specifics of phase-locked loop design using the phase/frequency detector, the problem will be dealt with using the following approach:

1) A general discussion of sampling phase-locked loops will be given which will display some of the differences between the true open-loop gain function, and the commonly used continuous approximation to the open-loop gain function.

2) With the sampling basics now developed, the transfer function for the phase/frequency detector will be found in some detail.

3) The phase/frequency detector transfer function is used to write an accurate impulse for the open-loop gain function. Given this function, a band-limited approximation of the open-loop gain function will be found using Z-transforms. The final band-limited expression can be used with conventional continuous transform (Laplace transforms) design methods for phase-locked loops.

Sampling Phase-Locked Loop Fundamentals

In contrast to the continuous mixer-type phase detector, the phase/frequency detector is a sampling phase detector. Phase error information is available at discrete time intervals which are spaced at exact intervals of T seconds, where T is the period of the reference frequency. The phase error at each instant is in the form of an impulse function whose area is proportional to the phase error. In reality, the impulses out of a phase-frequency detector such as the Motorola 4344 are of finite amplitude, but this fact is completely negligible in light of the RC filter time constants which follow the device. The phase detector output pulses may be mathematically viewed as ideal impulse functions of appropriate area. For the time being, the concept of the phase/frequency detector as an ideal impulse sampler will be deferred and developed momentarily.

The phase-frequency detector is modeled in Figure 1 as an ideal impulse sampler. The function H(s) represents some form of analog "hold" function such as the RC lowpass filters that customarily follow a phase-frequency detector. The function G(s) represents the normal loop filter transfer function.

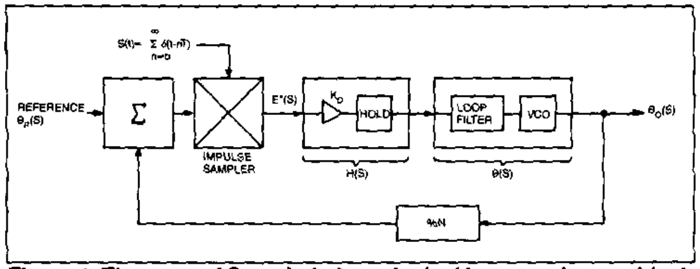

Figure 1. The general Sampled phase-locked loop employs an ideal Impulse Sampler which must be followed by some form of "hold" device. (HS).

Before jumping into the loop details, as an aside, consider the time function f(t) which is sampled by an ideal impulse sampler. The sampled time function can be written as in (1) where the asterisk (*) represents the time-sampled form of the function.

$$f^*(t) = f(t) \sum_{n=0}^{\infty} \delta(t - nT) \tag{1}$$

From Laplace platform and convolution theory, (1) may be written in Laplace transforms as in (2) where $F^*(s)$ and $f^*(t)$ are Laplace transform pairs. Note that F(s) and f(t) are also Laplace transform pairs.

$$F^*(s) = F(s) \star L\left\{\sum_{n=0}^{\infty} \delta(t - nT)\right\} \tag{2}$$

The ★ in (2) represents convolution in the frequency domain. Equation (1) may be used with the definition of the forward one-sided Laplace transform to give yet another interpretation of the time-sampled function form in the frequency domain.

$$F^*(s) = \int_0^{\infty} f^*(t) \exp(-sT)dt \tag{3}$$

$$= \int_0^{\infty} f(t) \sum_{n=0}^{\infty} \delta(t-nT) \exp(-sT)dt$$

$$F^*(s) = \sum_{n=0}^{\infty} \int_0^{\infty} \delta(t-nT) \exp(-sT)dt \tag{4}$$

$$F^*(s) = \sum_{n=0}^{\infty} f(nT) \exp(-sT). \quad (5)$$

Equation (5) is actually the defining relationship for the Z-transform of f(t). This fact will be used later in this article for easy calculation of $F^*(s)$.

A very powerful relationship may be found by continuing the convolution calculation in (2). Since the convolution must be performed in the frequency domain, we must know the Laplace transform of the infinite series of impulse functions which are performing the sampling operation. Since

$$\delta(t) \leftrightarrow 1 \quad (6)$$

then

$$L\{\sum_{n=0}^{\infty}\delta(t-nT)\}=1+\exp(-sT)+\exp(-2sT)+$$
$$= \frac{1}{1-\exp(-sT)} \quad (7)$$

The convolution of (2) may be rewritten as

$$F^*(s)=F(s)\star\frac{1}{1-\exp(-sT)}$$
$$=\int\frac{F(u)\,du}{1-\exp(sT-uT)} \quad (8)$$
$$=\int\frac{F(Z)}{1-\exp(sT)/Z}\,\frac{dz}{TZ}$$

where $Z = \exp(uT)$ and $du = dZ/(ZT)$.

Taking this process one step further and using the Residue theorem, this integral may be evaluated as in (9).

$$=\frac{1}{T}\int\frac{F(Z)\,dZ}{Z-\exp(-sT)} \quad (9)$$

$$=\frac{1}{T}\sum_{n=-\infty}^{\infty}F(s+j\,n\,Ws) \qquad \text{where } Ws = 2\pi/T.$$

Equation (9) is a valuable result and although it involves an infinite summation, it may be used to evaluate $F^*(s)$.

These previous transform tools will appear much more valuable if we now return to a discussion of Figure 1. As in classical PLL analysis, the most expedient first step in the loop analysis is to solve for the error function $E^*(s)$. We may write

$$\begin{aligned} E^*(s) &= (\Phi r - \Phi o)^* \\ &= \Phi r^* - \Phi o^* \\ &= \Phi r^* - [\,E^*(s)\,H(s)\,G(s)\,]^* \end{aligned} \quad (10)$$

Those unfamiliar with sampled systems will find [4] particularly useful and easy to understand. As developed in Chapter 4 of [4], the sampling operation in (10) may be brought within the brakcets because E(s) is already a sampled function. The sampled error function is then given by

$$E^*(s) = \Phi r^* - E^*(s)\,HG^*(s) \quad (11)$$

We finally obtain the desired result for the sampled loop error function.

$$E^*(s) = \sum_{n=-\infty}^{\infty}\frac{\Phi r(s+j\,n\,Ws)/T}{1+HG^*(s)} \quad (12)$$

In most cases, $\Phi r(s)$ can be assumed to be effectively bandlimited and aliasing of noise products can be neglected. This gives some simplication to (12) as given in (13).

$$E^*(s) = \frac{\Phi r(s)/T}{1+HG^*(s)} \quad (13)$$

where $\Phi^*(s) \sim \Phi r(s)/T$.

The asterisks would be absent in classical analysis of a phase-locked loop which neglected sampling effects. With rare exception, most systems which use the phase/frequency detector have a small bandwidth compared to the reference frequency in order to obtain reasonably low "sampling spurs." For this reason, the higher order terms (terms other than n=0) in equation (9) can largely be ignored for low bandwidth situations. This is precisely why classical analysis ignoring sampling effects still provides excellent results in small bandwidth situations. As the loop bandwidth is increased compared to the reference frequency, however, the higher order terms cannot be ignored.

For large loop bandwidth situations (bandwidth > 0.1 F_{ref}), Z-transform techniques should be used to include the higher order effects. If there are true transport time delays within the loop, modified Z-transforms should be used. If the loop bandwidths remain small (say < 0.1 F_{ref}), bandlimited forms of the open-loop gain function may be found which very accurately describe sampling effects without resorting to Z-transform analysis. Equation (9) which is repeated below as equation (14) will provide the menu for arriving at a bandlimited form of the open-loop gain function including sampling effects. The continuous Laplace transform impulse response, Gol(s), must first be found. The continuous open-loop gain function will be re-expressed in terms of Z-transforms and the assumption of small loop bandwidth imposed. The final result will be the bandlimited form of the open-loop gain function with first order sampling effects.

$$F(Z) = \frac{1}{T}\sum_{n=-\infty}^{\infty}F(s+j\,n\,Ws) \quad (14)$$

It will be shown that the so called "divider time delay" appears during this step and is solely a result of sampling.

Impulse Response of the Phase/Frequency Detector

A simplistic equivalent circuit of the phase/frequency detector is provided in Figure 2. No attempt has been made here to describe the frequency discriminator mode of operation. In many applications, the phase detector remains in its linear range of operation, because although the phase error may be very large at the VCO, it is reduced by N at the phase detector.

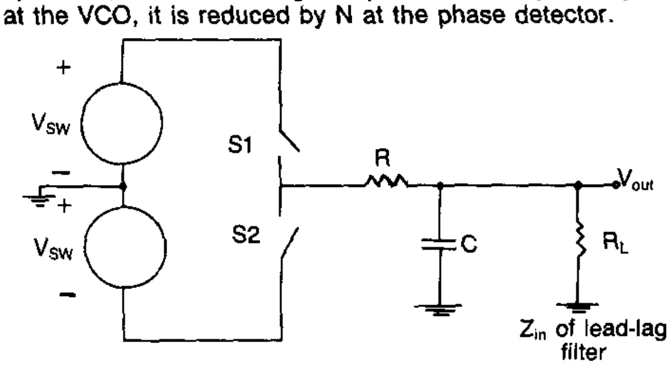

Figure 2

In steady-state operation, the switches in Figure 2 are open 95 percent of each reference period, only closing long enough to replenish the small discharge in capacitor C each reference period. (Since the 4344 type phase detector cannot resolve absolute time difference between the divider and reference pulse trains less than its own internal time delay, some built in offset is needed to avoid the detector's "dead zone" of operation at zero time difference between the two waveform trains.) During the period of frequency acquisition, the proper polarity switch is closed for a length of time which is defined by the time difference between the leading edge of the divide-by-N signal. This signal relationship is shown in Figure 3. The phase detector output pulse width is directly proportional to the phase error within the loop.

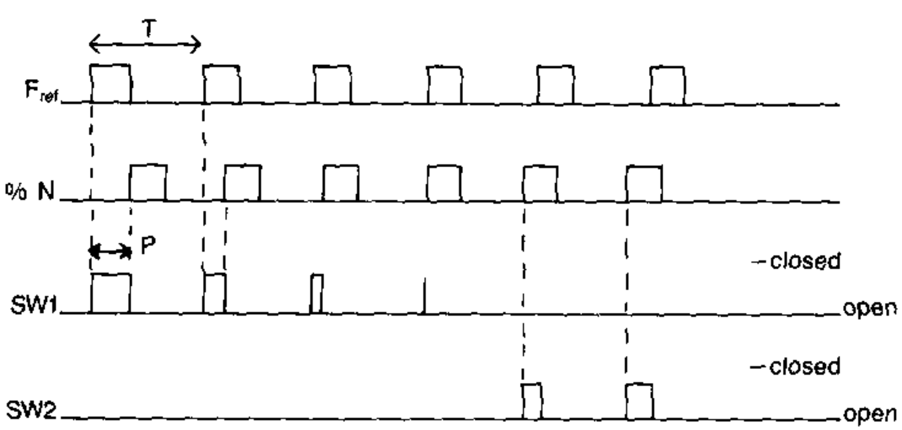

Figure 3

$$\Phi e = \frac{P}{T} 2\pi \tag{15}$$

or

$$p = \frac{T}{2\pi} \Phi e$$

The pulse widths out of the phase detector, p, are very small with respect to the reference period because a Type II loop is always used (zero steady-state phase error) and the VCO phase error is reduced by the divider ratio, N.

We are primarily interested in the impulse response of the phase detector/lowpass filter combination. The transfer function for the lowpass filter alone is given by (16).

$$FL(s) = \frac{R1}{R1 + R} \frac{1}{1 + s\,\tau 1} \tag{16}$$

where $\tau 1 = \dfrac{R1\ R\ C}{R1 + R}$

A typical pulse response of the circuit in Figure 2 is presented in Figure 4. The rising edge is very linear because $p << \tau 1$. The output voltage, V_{out}, can be easily found from Figure 4. The output voltage during the next reference period is given by (17).

$$V_{out}(t) = V_o \exp(-t/\tau 1) + \frac{R1}{R1 + R} V_{sw}(1 - \exp(-t/\tau 1)) \tag{17}$$

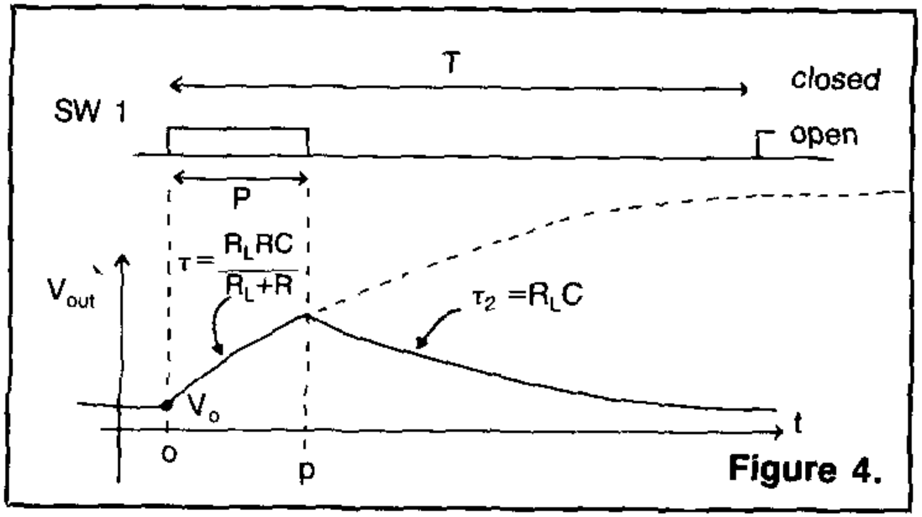

Figure 4.

for $0 < t <= p$

$V_{out}(t) = V_{out}(p)\ \exp(-t/\tau 2)$ for $p < t < T$.

In a properly designed Type II loop under normal linear operation, $p << \tau 1$ and equation (17) may be written in a more simple form without the first exponentials.

$$V_{out}(t) \sim V_o + \frac{R1}{R1 + R} V_{sw}(t/\tau 1) \tag{18}$$

for $0 < t <= p$

$V_{out}(t) = V_{out}(p)\ \exp(-t/\tau 2)$ for $p < t <= T$.

It is important to note once more that V_o represents the initial capacitor voltage due to any previous phase sample, Φe.

The Laplace transform transfer function can be found by using the above equations directly.

$$V_{out}(s) = L\{(V_o + \frac{R1}{R1 + R} V_{sw} \frac{p}{\tau 1})\ \exp(-t/\tau 2)\}$$

$$= \frac{V_o}{s + 1/\tau 1} + \frac{R1\ V_{sw}\ \Phi e\ T}{(R1+R)\ \tau 1\ 2\ \pi\ (s+1/\tau 2)} \tag{19}$$

where the phase error is $\Phi e = 2\pi\ p/T$.

As stated earlier, V_o is a direct result of earlier samples of the phase error, Φe. Using this fact, it is possible to show that the output voltage as a function of the input phase error is given by (20).

$$\frac{V_{out(s)}}{\Phi e(s)} = \frac{Kd\ T}{(R1+R)/R1+sRC} \frac{1}{1-\exp(-sT)\ \exp(-T/\tau 2)} \tag{20}$$

where $Kd = V_{sw}/(2\pi)$

This is the final result for the phase detector impulse response. The factor T is a direct result of the sampling operation. Notice that if $T/\tau 2$ is not large, the second multiplicative factor cannot be ignored. In this case, the output voltage is a function of the present phase detector error as well as the previous error samples and Z-transform analysis is required. Since the intent of this analysis has been to eventually arrive at a band-limited form of the open-loop gain function, in that vein, $T/\tau 2$ will be assumed to be >> 1 such that Z-transform analysis will not be required in the final end result.

Derivation of the Continuous Band-Limited Open-Loop Gain Function with Sampling

The phase-frequency detector is always used in a Type II phase-locked loop in order to realize reasonable spurious performance and tuning range. In order to simplify the mathematics involved, however, an example using a Type I system will be used. Our approach will be to calculate Gol(Z) and compare it to the continuous form of Gol(s). This will reveal the effects of sampling upon the otherwise continuous open-loop gain expression.

In order to make any connection between an impulse sampled system and a continuous system, the sampled loop must have some form of "hold" device which effectively converts the phase detector impulse functions into smooth time waveforms which have a finite width in time, and a finite height. If the "hold device" is not present, the loop *must* be analyzed as a sampled system. No equivalent continuous system would exist for that case.

The "hold" device may be as simple as an RC lowpass filter, or as complicated as a true 0-order sample/hold. Consider a continuous Type I phase-locked loop with a low pass "hold" as given in (21).

$$\mathrm{Gol}(s) = \frac{Kd\ Kv}{N\ s} \rightarrow \frac{Wn}{s}\ \frac{1}{1+s\tau} \tag{21}$$

where
Kd=phase detector gain
Kv=VCO sensitivity
N =feedback divider ratio
τ =low pass filter time constant representing the "hold."

As shown earlier, the sampling effects upon a continuous function may be included by taking the Z-transform of the time function provided that the continuous function is correct of course. In the previous section, it was shown that the impulse response of the phase/frequency detector followed by a simple RC lowpass filter is given by (20). Therefore, assuming that T >> $\tau 2$ in equation (20), equation (21) must be multiplied by T to have proper form. Using a table of Z-transforms, equation (21) may be easily converted into Gol(Z).

$$\frac{Wn\ T}{s(1+s\tau)} \rightarrow Wn\ (1-\exp(-t/\tau))\ T \tag{22}$$

$$\rightarrow \frac{Wn\ Z\ T}{Z-1} - \frac{Wn\ Z\ T}{Z-A}$$

Collecting terms in (22), the Z-transform for the Type I system is simply given by (23).

$$\mathrm{Gol}(Z) = \frac{Wn\ Z\ (1-A)\ T}{(Z-1)(Z-A)} \tag{23}$$

We can effectively remove any significant effects of the "hold" device upon the system by allowing $\tau \rightarrow 0$, i.e., $A \rightarrow 0$. This is equivalent to making the RC filter time constant negligible compared to the reference period, T. In the limit as $A \rightarrow \Phi$,

$$\mathrm{Gol}(Z) = \frac{Wn\ Z\ T}{(Z-1)(Z-0)} = \frac{Wn\ T}{(Z-1)} \tag{24}$$

Equation (24) may be expressed in terms of the more familiar complex frequency, s, by noting that Z= exp(sT). Doing so, we obtain (25).

Number of harmonic terms included in summation 5
Loop reference frequency 10000
Typ 1 loope Wn 2000
Loop LPF time constant, nsec 5000
APPROXIMATION TO SAMPLED OPEN-LOOP GAIN FUNCTION

	Summation of Gol Terms		Continuous Gol Exp(-ST/2)	
F	Gol, dB	Ang, Deg		
100	10.06	-91.20	10.06	-91.98
150	6.53	-91.80	6.54	-92.97
200	4.03	-92.40	4.04	-93.96
300	0.51	-93.61	0.51	-95.94
400	-2.00	-94.81	-1.98	-97.92
500	-3.94	-96.03	-3.92	-99.90
700	-6.89	-98.47	-6.85	-103.86
1000	-10.03	-102.21	-9.95	-109.80
1500	-13.65	-108.72	-13.47	-119.70
2000	-16.28	-115.74	-15.98	-129.60
3000	-20.07	-132.17	-19.52	-149.38
4000	-22.55	-153.18	-22.05	-169.16

Number of harmonic terms included in summation 10
Loop reference frequency 10000
Type I loop Wn 2000
Loop LPF time constatn, nsec 5000
APPROXIMATION TO SAMPLED OPEN-LOOP GAIN FUNCTION

	Summation of Gol Terms		Continuous Gol Exp(-ST/2)	
F	Gol, dB	Ang, Deg		
100	10.06	-91.46	10.06	-91.98
150	6.54	-92.19	6.54	-92.97
200	4.04	-92.93	4.04	-93.96
300	0.51	-94.39	0.51	-95.94
400	-1.98	-95.86	-1.98	-97.92
500	-3.92	-97.33	-3.92	-99.90
700	-6.84	-100.30	-6.85	-103.86
1000	-9.94	-104.79	-9.95	-109.80
1500	-13.45	-112.49	-13.47	-119.70
2000	-15.92	-120.54	-15.98	-129.60
3000	-19.24	-138.14	-19.52	-149.38
4000	-21.15	-158.08	-22.05	-169.16

Number of harmonic terms included in summantion 5
Loop reference frequency 1000
Type II loop Wn 2000
Loop damping factor, eta .707
APPROXIMATION TO SAMPLED OPEN-LOOP GAIN FUNCTION

	Summation of Gol Terms		Continuous Gol Exp(-ST/2)	
F	Gol, dB	Ang, Deg		
100	20.95	-156.41	20.90	-158.03
150	14.77	-147.09	14.67	-149.29
200	10.75	-139.68	10.60	-142.34
300	5.68	-129.45	5.46	-132.82
400	2.40	-123.29	2.22	-127.29
500	0.16	-119.51	-0.11	-124.14
700	-3.13	-115.77	-3.41	-121.69
1000	-6.45	-114.59	-6.72	-122.49
1500	-10.16	-117.17	-10.37	-128.23
2000	-12.79	-122.18	-12.92	-136.02
3000	-16.50	-136.40	-16.49	-153.68
4000	-18.84	-155.59	-19.03	-172.38

Number of harmonic terms included in summation 10
Loop reference frequency 1000
Type II loop Wn 1000
Loop LPF time constant, nsec 5000
APPROXIMATION TO SAMPLED OPEN-LOOP GAIN FUNCTION

	Summation of Gol Terms		Continuous Gol Exp(-ST/2)	
F	Gol, dB	Ang, Deg		
100	10.70	-139.13	10.60	-140.36
150	5.60	-128.39	5.46	-129.85
200	2.38	-121.69	2.22	-123.33
300	-1.73	-114.49	-1.93	-116.51
400	-4.46	-111.20	-4.67	-113.64
500	-6.51	-109.71	-6.72	-112.59
700	-9.53	-109.20	-9.75	-112.99
1000	-12.68	-111.03	-12.90	-116.22
1500	-16.21	-116.62	-16.46	-123.99
2000	-18.67	-123.59	-18.98	-132.82
3000	-21.97	-139.95	-22.53	-151.53
4000	-23.05	-158.98	-25.06	-170.77

$$Gol(s) = \frac{1}{j2}\,\frac{Wn\,\exp(-sT/2)T}{\sin(wT/2)} \tag{25}$$

For frequencies which are small compared to the reference frequency, $F_{ref} = 1/T$, the sin(x) ~ x approximation may be made, reducing (25) to finally (26). This is equivalent to the initial premise that the loop bandwidth is much less than the reference frequency.

$$Gol(s) = \frac{Wn\,\exp(-sT/2)}{s} \tag{26}$$

The final result for the bandlimited form of the open-loop gain function is given in (27). This expression includes the first order sampling effects. The appearance of the so-called time delay exponential occurred without introducing any transport time delay whatsoever, only the sampling effects.

$$Gol(s) = \frac{\exp(-sT/2)\,Wn}{s} \tag{27}$$

Generalizing, first order sampling effects for phase-locked loops which have a small percentage bandwidth compared to the reference frequency can be analyzed using classical Laplace transform methods provided the new "delay term" is included in the phase detector transfer function.

$$Kd\,\exp(-sT/2) \tag{28}$$

Further Proof

As further proof of our result above, we may compare this result with that obtained using (9). Only the first few terms of the infinite summation in (9) will be included. The computer program and sample run appear in Appendix I. Notice that the inclusion of the added exponential term of (28) with the normal Type I open-loop gain results in very good agreement between the two mathematical models for frequencies well within the closed-loop bandwidth. The phase of the open-loop gain function would be very inaccurate had the exponential term been left out. As the loop bandwidth increases with respect to the reference frequency, the approximation shows more and more deviation from the true open-loop gain calculated by (9). [Note that for all cases, $T/\tau 1 >> 1$ has been assumed with loop bandwidth $<< F_{ref}$.]

In order to be complete, the same calculation was performed for the Type II phase-locked loop with a phase-frequency detector and small RC lowpass filter "hold." The continuous form of the open-loop gain function is given in (29) where T is due to the phase detector transfer function.

$$Gol(s) = \frac{Kd\,T}{1 + s\tau}\,\frac{1+s\,\tau 2}{s\tau 1}\,\frac{Kv}{N\,s} \tag{29}$$

Our approximation to G*ol(s) is found using equation (28).

$$G^*ol(s) \sim \exp(-sT/2)\,\frac{Kd}{1+s\tau}\,\frac{1+s\tau 2}{s\tau 1\,N}\,\frac{Kv}{s} \tag{30}$$

The true function G*ol(s) is found again from substituting equation (29) into (9).

$$G^*ol(s) = \frac{1}{T}\sum_{n=-\infty}^{\infty} \frac{Kd\,T}{1+u\tau}\,\frac{1+u\,\tau 2}{u\,\tau 1}\,\frac{Kv}{N\,u} \tag{31}$$

where u = s + jnWs

Reiterating, the T following Kd is due to the phase detector transfer function, (20), whereas the 1/T is due to the leading coefficient in (14).

A second computer program and sample run are provided in Appendix I for this Type II phase-locked loop case. Once again, the bandlimited gain expression in (30) closely approximates the true gain function (31) for frequencies well within the closed-loop bandwidth.

In concluding this article, several statements stand out.

- In order to make any correlation between sampled and continuous systems, some form of analog "hold" device impulse response of the hold device is the interpolating waveform between sample points in the time domain.
- The appearance of sampling in any loop with the accompanying "hold" device, causes an exponential phase term to appear, $\exp(-sT/2)$.
- Sampling effects in small percentage bandwidth loop (wrt. F_{ref}) may be quite accurately described by the normal continuous open-loop gain function provided that the additional exponential term is included.

Sampling effects cause the appearance of the exponential delay-like term whenever quantities within the loop are only available at dicrete instants in time. Digital dividers within the feedback loop in an otherwise continuous loop will still cause the exponential term to appear. If digital feedback dividers and a digital phase/frequency detector are used within the phase-locked loop together, only one $\exp(-sT/2)$ term results (the continuous first order approximation remains unchanged). True transport delay within the loop is accounted for by another exponential delay term. (Here, true transport delay refers to delay through op-amps and propagation delay refers to delays through other components, including dividers. These quantities are available in component data books.)

Analysis of the phase/frequency detector in a phase-locked loop can be considerably more complex than the usual continuous analysis which is generally employed. If the loop bandwidth is, say, $< F_{ref}/40$, the loop can be considered continuous for all practical purposes. For higher percentage bandwidths, attention should be given to the added "delay-like" term shown in equation (28) and care should be given to insure that $T/\tau 2$ is > 3 in equation (20). [If $T/\tau 2 < 3$, another integrator in the form of a time variable filter is created which makes the situation much more complex. Of course, keeping $T/\tau 2 > 3$ will result in higher spurs and notch filtering will undoubtably be required. The time variable filter increases the gain within the loop bandwidth and adds substantial phase as well which can easily lead to instability. For best results, choose $T/\tau 2 > 3$.]

Although the cautious aspects of sampled phase-locked loop design have been brought out for the phase/frequency detector, sampled systems harbor much more capability than first glance indicates. For instance, a Type I sampled loop which employs a zero order sample and hold rather than the phase/frequency detector will theorectically perform phase-lock in only one sample period! An ideal Type II phase-locked loop with a zero order sample and hold phase detector is capable of performing phase-lock in only two sample periods. These speed-optimized phase-locked loops must be analyzed using Z-transforms.[5]

References

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3. *RF Design,* "Divider Delay: The Missing PLL Analysis Ingredient," Stan Goldman, March/April 1984, pp. 58A-66A.

4. *Digital Control of Dynamic Systems,* Gene F. Franklin, J. David Powell, Addison-Wesley.

5. *Microwaves & RF,* "Sampling Phase-Locked Loops for Frequency Synthesis," J.A. Crawford (to be published).

Appendix I

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FILE: SAMPLL   ?1                                        PAGE 001

10 '*********************
20 '
30 '    Comparison of    Sum[  F(s + jnWs) ]  with Z-transforms
40 '
50 '*********************
60 DIM PTS(20),MAG(20),ANG(20)
62 PI=3.141592654#
70 FOR I%=1 TO 17
80      READ PTS(I%)
90 NEXT I%
100 DATA 100,150,200,300,400,500,700,1000,1500,2000,3000,4000,5000,7000
110 DATA 8000,9000,10000
120 '
130 INPUT "NUMBER OF HARMONICS TO INCLUDE ";NHARM%
135 INPUT "INPUT REFERENCE RATE, HZ  ";FREF
136 INPUT "INPUT THE TYPE I LOOP    WN  ";WN
137 INPUT "INPUT THE LOW-PASS FILTER   TIMECONSTANT, NSEC";TAU
138 TAU=TAU*9.999999E-10
140
150 FOR I%=1 TO 17
160     SUMR=0
170     SUMI=0
180     FOR J%=1 TO NHARM%
190             F=PTS(I%) + J%*FREF
200             GOSUB 1000
210             SUMR=SUMR + FR
220             SUMI=SUMI + FI
230             F=PTS(I%) - J%*FREF
235             GOSUB 1000
240             SUMR=SUMR + FR
250             SUMI=SUMI + FI
260     NEXT J%
270     F=PTS(I%)
280     GOSUB 1000
290     SUMR=SUMR + FR
300     SUMI=SUMI + FI
310     MAG(I%)=4.34294*LOG( SUMR^2 + SUMI^2 )
320     ANG(I%)=ATN(SUMI/(SUMR + 1E-08))
330     IF SUMR<0 THEN ANG(I%)=ANG(I%)-PI
333     ANG(I%)=ANG(I%)*180/PI
340 NEXT I%
350 PRINT CHR$(26)
351 LPRINT "Number of harmonic terms included in summation  ";NHARM%
352 LPRINT
353 LPRINT "Loop reference frequency  ";FREF
354 LPRINT
355 LPRINT "Type I  loop   Wn  ";WN
356 LPRINT
357 LPRINT "Loop   LPF  time constant, nsec  ";TAU*1E+09
358 LPRINT
360 LPRINT "APPROXIMATION TO SAMPLED OPEN-LOOP GAIN FUNCTION"
370 LPRINT "                                                      Continuous Gol"
371 LPRINT "               Summation of  Gol Terms                 Exp( -ST/2 )"
372 LPRINT
380 LPRINT "F                 Gol,dB        Ang,Deg  "
390 LPRINT "================================================"
400 FOR I%=1 TO 17
401     F=PTS(I%)
402     GOSUB 1000
403     NORGAIN=4.34294 * LOG( FR^2 + FI^2 )
404     NORANG=B - W*.5/FREF
410     LPRINT USING "#######      ####.##      ####.##            ####.##     ####
.##";PTS(I%),MAG(I%),ANG(I%),NORGAIN,NORANG*180/PI
```

```
420 NEXT I%
430 STOP
1000 '***************
1010 '   OPEN LOOP GAIN FUNCTION, CONTINUOUS
1020 '***************
1030 W=F*2*PI
1040 A=WN/( W*SQR( 1 + (W*TAU)^2 ) )
1050 B=-PI*.5 - ATN(W*TAU)
1060 FR=A*COS(B)
1070 FI=A*SIN(B)
1080 RETURN
```

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```
10 '*********************
20 '
30 '     Comparison of    Sum[  F(s + jnWs) ]  with Z-transforms
40 '
50 '*********************
60 DIM PTS(20),MAG(20),ANG(20)
70 PI=3.141592654#
80 FOR I%=1 TO 17
90      READ PTS(I%)
100 NEXT I%
110 DATA 100,150,200,300,400,500,700,1000,1500,2000,3000,4000,5000,7000
120 DATA 8000,9000,10000
130 '
140 INPUT "NUMBER OF HARMONICS TO INCLUDE ";NHARM%
150 INPUT "INPUT REFERENCE RATE, HZ  ";FREF
160 INPUT "INPUT THE TYPE II LOOP    WN  ";WN
170 INPUT "INPUT THE LOOP DAMPING FACTOR  ";ETA
180 INPUT "INPUT THE LOW-PASS FILTER   TIMECONSTANT, NSEC";TAU
190 TAU=TAU*9.999999E-10
200 '
210 FOR I%=1 TO 17
220     SUMR=0
230     SUMI=0
240     FOR J%=1 TO NHARM%
250             F=PTS(I%) + J%*FREF
260             GOSUB 680
270             SUMR=SUMR + FR
280             SUMI=SUMI + FI
290             F=PTS(I%) - J%*FREF
300             GOSUB 680
310             SUMR=SUMR + FR
320             SUMI=SUMI + FI
330     NEXT J%
340     F=PTS(I%)
350     GOSUB 680
360     SUMR=SUMR + FR
370     SUMI=SUMI + FI
380     MAG(I%)=4.34294*LOG( SUMR^2 + SUMI^2 )
390     ANG(I%)=ATN(SUMI/(SUMR + 1E-08))
400     IF SUMR<0 THEN ANG(I%)=ANG(I%)-PI
410     ANG(I%)=ANG(I%)*180/PI
420 NEXT I%
430 PRINT CHR$(26)
440 LPRINT "Number of harmonic terms included in summation  ";NHARM%
450 LPRINT
460 LPRINT "Loop reference frequency  ";FREF
470 LPRINT
480 LPRINT "Type II  loop   Wn  ";WN
490 LPRINT
500 LPRINT "Loop damping factor, Eta  ";ETA
510 LPRINT
520 LPRINT "Loop   LPF  time constant, nsec  ";TAU*1E+09
530 LPRINT
540 LPRINT "APPROXIMATION TO SAMPLED OPEN-LOOP GAIN FUNCTION"
550 LPRINT "                                                    Continuous Gol"
560 LPRINT "                  Summation of  Gol Terms             Exp( -ST/2 )"
570 LPRINT
580 LPRINT "F                Gol,dB          Ang,Deg  "
590 LPRINT "________________________________________________"
600 FOR I%=1 TO 17
610     F=PTS(I%)
620     GOSUB 680
630     NORGAIN=4.34294 * LOG( FR^2 + FI^2 )
640     NORANG=B - W*.5/FREF
650     LPRINT USING "#######      ####.##       #####.##          ####.##      ####.##";PTS(I%),MAG(I%),ANG(I%),NORGAIN,NORANG*180/PI
660 NEXT I%
670 STOP
680 '***************
690 '   OPEN LOOP GAIN FUNCTION, CONTINUOUS
700 '***************
710 W=F*2*PI
720 A= (WN/W)^2 * SQR(  (1+ (2*ETA*W/WN)^2) / ( 1 + (W*TAU)^2 ) )
730 B=-PI + ATN(2*ETA*W/WN) - ATN(W*TAU)
740 FR=A*COS(B)
750 FI=A*SIN(B)
760 RETURN
```