Coaxial Line and 50 Ohms

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1. Introduction

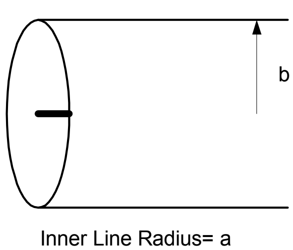
If you have ever wondered why standard coaxial cable has a characteristic impedance of 50 ohms, you have come to the right place. In this brief memo, we will see that this is a direct consequence of the fact that most coaxial cable is made using copper conductors.

2. Getting Started

In the discussion that follows, it will be assumed that the coaxial line in question has a perfect lossless dielectric with a relative permittivity[1] of 2.25. The only losses present in the cable are then conductive losses due to the copper center conductor and shield of the coax line. Since the dielectric is assumed to be lossless, the distributed conductance G is zero. It can be shown that loss through the coax line can be minimized for a particular choice of characteristic impedance Z_o. The related loss is characterized by the real part of the propagation constant, α.

Maxwell's equations can be used to obtain expressions for the distributed inductance and capacitance of the coaxial line.

Figure 1 Coaxial Line Dimensions



For a static current I flowing in the center conductor, the magnetic field can be easily calculated from

[1] Relative dielectric constant of polypropylene, a popular low-cost coaxial dielectric material

$$\oint \vec{B} \bullet d\vec{l} = \mu_o I \qquad (1)$$

from which

$$B = \frac{\mu_o I}{2\pi r} \qquad (2)$$

The inductance per unit length can be calculated as

$$L = \frac{\Phi}{I} = \frac{1}{I}\int_a^b dr \int_0^1 dz \frac{\mu_o I}{2\pi r} = \frac{\mu}{2\pi} ln\left(\frac{b}{a}\right) \qquad H/m \qquad (3)$$

For the capacitance per unit length, we know that

$$C = \frac{Q}{V} \qquad (4)$$

and

$$\oint \vec{E} \bullet d\vec{S} = \frac{Q}{\varepsilon} \qquad (5)$$

The latter equation combined with the symmetries present in the coaxial cable result in

$$E = \frac{Q}{2\pi\varepsilon r} \qquad (6)$$

We also know that

$$V = \int \vec{E} \bullet d\vec{l} = \int_a^b \frac{Q}{2\pi\varepsilon r} = \frac{2\pi\varepsilon Q}{ln\left(\frac{b}{a}\right)} \qquad (7)$$

Using this result in (4) results in the distributed capacitance being given by

$$C = \frac{2\pi\varepsilon}{ln\left(\frac{b}{a}\right)} \qquad (8)$$

The complex propagation constant is given by[2]

[2] E.C. Jordan, K.G. Balmain, , *Electromagnetic Waves and Radiating Systems, 2nd Edition, 1968, Prentice-Hall,* Equ. (7-110)

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)} \quad (9)$$

which simplifies in the lossless dielectric case to

$$\alpha \approx \frac{R}{2Z_o} \quad (10)$$

The resistance per unit length of copper is given as a function of frequency f by

$$R(f) = 4.16 \cdot 10^{-8}\sqrt{f}\left(\frac{1}{a}+\frac{1}{b}\right) \quad (11)$$

and with $Z = \sqrt{L/C}$, upon substitution the final result for α is

$$\alpha = \frac{3.467 \cdot 10^{-10}\sqrt{f}\left(\frac{1}{a}+\frac{1}{b}\right)}{\sqrt{\varepsilon_r}\ln\left(\frac{b}{a}\right)} \quad (12)$$

As clearly shown in (12), the minimization of α comes down to choices for the radii a and b, specifically the ratio of the two parameters. In order to minimize α, assume that we arbitrarily set b=1.0 and set the derivative of α with respect to a equal to zero. In doing so, let

$$K = \frac{3.467 \cdot 10^{-10}\left(\frac{1}{a}+1\right)}{0-\ln(a)}\frac{1}{\sqrt{\varepsilon_r}} \quad (13)$$

From this form,

$$\frac{dK}{da} = 0 = \frac{\frac{\ln(a)}{a^2}+\left(1+\frac{1}{a}\right)\frac{1}{a}}{\left[\ln(a)\right]^2} \quad (14)$$

Clearly, it must be true that 0 < a < 1, and a must satisfy the equation

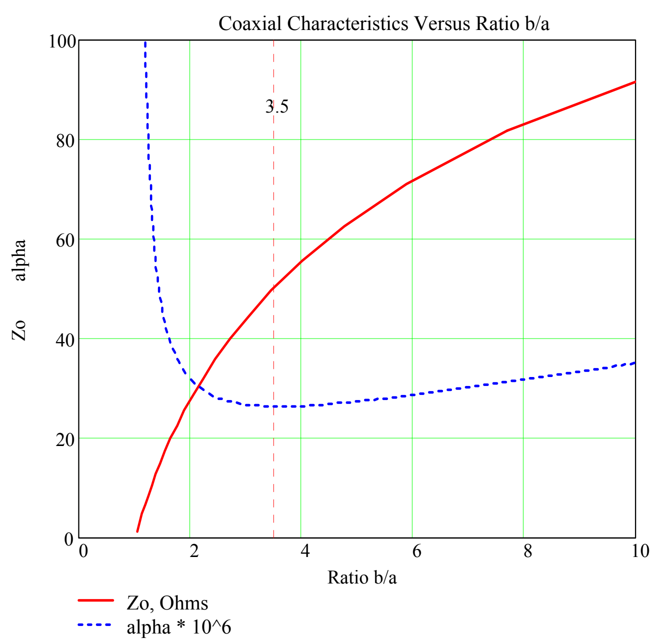
$$\ln(a) + a + 1 = 0 \quad (15)$$

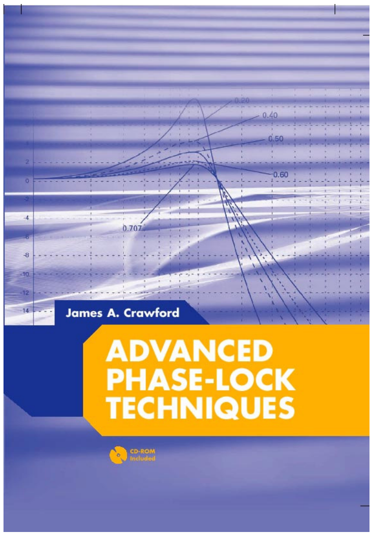
The value of a that satisfies this final constraint is 0.278465 or equivalently, b/a= 3.591156 and

$$\begin{aligned} Z_o &= \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{b}{a}\right) \\ &= \frac{60}{\sqrt{\varepsilon_r}}\ln\left(\frac{b}{a}\right) \\ &= 51.14 \quad Ohms \end{aligned} \quad (16)$$

in the case where ε_r= 2.25. Fortunately, the loss is not a strong function of the ratio b/a and it suffices to choose the convenient value 3.5 which leads to a characteristic impedance value of 50.11 ≈ 50 Ohms. Alpha and the characteristic impedance are plotted versus the ratio b/a in Figure 2.

Figure 2 Coaxial Parameters Versus (b/a) at 600 MHz





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3	*Fundamental Limits* A detailed discussion of the many fundamental limits that PLL designers may have to be attentive to or else never achieve their lofty performance objectives, e.g., Paley-Wiener Criterion, Poisson Sum, Time-Bandwidth Product.	38
4	*Noise in PLL-Based Systems* An extensive look at noise, its sources, and its modeling in PLL systems. Includes special attention to $1/f$ noise, and the creation of custom noise sources that exhibit specific power spectral densities.	66
5	*System Performance* A detailed look at phase noise and clock-jitter, and their effects on system performance. Attention given to transmitters, receivers, and specific signaling waveforms like OFDM, M-QAM, M-PSK. Relationships between EVM and image suppression are presented for the first time. The effect of phase noise on channel capacity and channel cutoff rate are also developed.	48
6	*Fundamental Concepts for Continuous-Time Systems* A thorough examination of the classical continuous-time PLL up through 4^{th}-order. The powerful Haggai constant phase-margin architecture is presented along with the type-3 PLL. Pseudo-continuous PLL systems (the most common PLL type in use today) are examined rigorously. Transient response calculation methods, 9 in total, are discussed in detail.	71
7	*Fundamental Concepts for Sampled-Data Control Systems* A thorough discussion of sampling effects in continuous-time systems is developed in terms of the z-transform, and closed-form results given through 4^{th}-order.	32
8	*Fractional-N Frequency Synthesizers* A historic look at the fractional-N frequency synthesis method based on the U.S. patent record is first presented, followed by a thorough treatment of the concept based on Δ-Σ methods.	54
9	*Oscillators* An exhaustive look at oscillator fundamentals, configurations, and their use in PLL systems.	62
10	*Clock and Data Recovery* Bit synchronization and clock recovery are developed in rigorous terms and compared to the theoretical performance attainable as dictated by the Cramer-Rao bound.	52