The Effects of Small Contaminating Signals in Nonlinear Elements Used in Frequency Synthesis and Conversion

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Abstract—Many electronic systems use nonlinear elements to add or subtract two frequencies or to multiply or divide a frequency by an integer. Some level of contamination by small undesired signals is always present and the ability to predict the effects produced by their passage through the nonlinear elements is important in analyzing system performance. These effects can often be predicted, for frequency mixing (addition and subtraction), multiplication and division, by decomposition of the contaminating signal into equivalent AM and FM sidebands whose effects are more easily estimated. One important effect that occurs in frequency division is a sampling process which translates the frequencies of the interfering signals. A method for predicting these effects is explained and experimental results, demonstrating the application and applicability of the method, are reported.

I. Introduction

MANY ELECTRONIC SYSTEMS, including superheterodyne radio receivers, use inherently nonlinear devices. Mixers, which are used in frequency conversion, typically have a nonlinear response to the stronger input, the local oscillator (LO). Frequency multipliers and dividers, used along with mixers in frequency synthesis, are nonlinear, and their use is growing as synthesis becomes ever more common in electronic equipment. The input that undergoes a nonlinear transfer in these devices is usually intended to be of a single frequency, or to consist of a single frequency and its harmonics, so the desired product can be isolated by filtering from other products generated in the nonlinearity. In practice, other, relatively small, signals (we use the term "signal" to include even noise components) will accompany the desired input because of inherent noise, stray coupling, and the imperfections of other processes used to generate the desired input signal. It is, therefore, important to be able to predict how these contaminating signals will pass through the nonlinearity so their effects on the system can be analyzed and bounds can be placed on their allowed levels.

A. Nonlinear Elements in a System

A system which employs many nonlinear devices where the effects of small contaminating signals are important is illustrated in Fig. 1. The system phase locks a voltage-controlled oscillator (VCO) to a reference frequency; this might represent a frequency synthesizer at some particular control setting. Small contaminating signals are likely to be present at A as a result of the digital circuitry employed in the generation of that signal. These will be in the form of both modulation sidebands and additional signals, which we will term single sidebands. Other contaminating signals will be present at E due to undesired waveforms on the tuning-voltage input and power-supply lines. In addition, undesired components will be generated in the mixing process. Once these undesired components have been identified, the following important questions arise. What happens to the level of the contaminating signals at A as they pass through the frequency multiplier? How are they affected in passing through the mixer? How are the resulting contaminants at C, plus those which come from the VCO or are generated in the mixer, affected in passing through the frequency divider and the phase detector to D?

B. Plan of the Paper

This paper proposes a method for analyzing the effect of these small contaminating signals based on characteristics of the nonlinearities which are often known, can be easily obtained, or can be reasonably estimated. After a general description of the method, its applications to mixers, to multipliers and to dividers are separately discussed. Then some experimental results are presented in order to illustrate the method, to confirm its usefulness and to show some limitations.

C. Basic Description of the Components

Before the description of the analysis method and experimental results begins, a brief tutorial review of the major components to be discussed, the frequency mixer, the frequency multiplier, the frequency divider and the phase detector, will be given. Those who are familiar with the operation of these components should proceed to the next section.

1) Mixers: Mixers are used to obtain a signal whose frequency is the sum or difference of the frequencies of two input signals, the RF and the LO signals. Mixers are commonly used in superheterodyne radios to translate the received RF signal to a relatively narrow band about the fixed intermediate frequency (IF), thus enabling the signal to be processed in fixed-tuned elements regardless of its original RF. The RF is chosen by tuning the LO to a frequency which translates the desired RF signal to the IF passband while translating other RF signals to frequencies which do not fall into the IF passband, and are therefore rejected.

The frequency translation in most mixers occurs because the LO and RF signals exist in a nonlinear element or elements. The second-order term in the nonlinearity produces a signal

$$V_2 = K_2(V_L \cos\theta_L + V_R \cos\theta_R)^2 \tag{1}$$

$$= K_2[\tfrac{1}{2}V_L^2(1+\cos 2\theta_L) + V_L V_R \cos(\theta_L - \theta_R) + V_L V_R \cos(\theta_L + \theta_R) + \tfrac{1}{2}V_R^2(1+\cos 2\theta_R)] \tag{2}$$

where V_L and V_R are the amplitudes of the LO and RF

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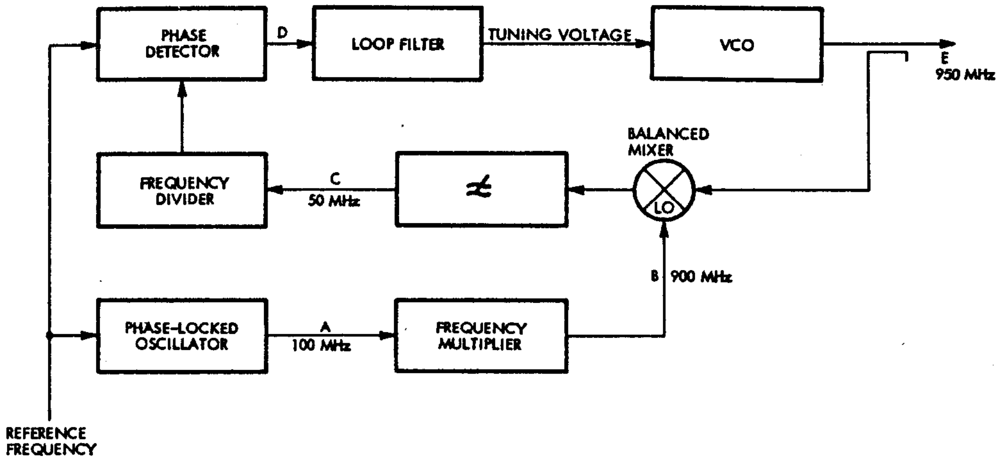

Fig. 1. A system which employs many nonlinear devices where the effects of small contaminating signals are important. Frequencies of the desired signals are indicated.

voltages,

$$\theta_L = \omega_L t + \alpha_L$$
$$\theta_R = \omega_R t + \alpha_R$$

and ω_L and ω_R are the LO and RF frequencies.

The second and third terms in (2) are at the difference and sum frequencies, respectively, and either may be chosen as the IF. Other orders of nonlinearity can also produce these same frequencies so that, in general, the coefficients of the various terms for the total response contain higher powers of V_L and V_R. The mixer is generally operated with V_R small enough that the lowest power term in V_R predominates but V_L is generally large enough that many other powers of V_L are significant. The total sum and difference frequency terms can therefore be expressed as

$$V_{2T} \approx K(V_L) \cdot V_R[\cos(\theta_L - \theta_R) + \cos(\theta_L + \theta_R)]. \quad (3)$$

Here, $K(V_L)$ is a function of the LO signal level and represents the conversion gain of the mixer. It rises with V_L and approaches a constant (less than one) at high levels of V_L, where the mixer is usually operated to maximize conversion gain. The mixer thus has a linear transfer function from RF to IF (except for the frequency translation) but is quite nonlinear in its relationship to the LO. If the LO is a pure, constant-amplitude sinusoid, the IF signal will look like the RF signal except for frequency translation, possibly some phase shift, and a constant attenuation. Two RF signals which differ by 10 dB will produce two sum or difference frequency signals which differ by 10 dB and whose frequencies have each been translated by the LO frequency. The same cannot be said about two LO signals. Other signals are also produced due to the linear and higher-order nonlinear responses of the mixer. These are signals at all frequencies given by

$$f = mf_R + nf_L \quad (4)$$

where m and n are positive or negative integers. The undesired responses, including leakage of the input signals (m or n equal zero), are often reduced by balanced design which generates several sets of desired signals and combines them in such a manner as to cancel some undesired signals. Balanced mixers have three ports, generally called LO, RF, and IF, and the signals at these ports are similarly identified (Fig. 2).

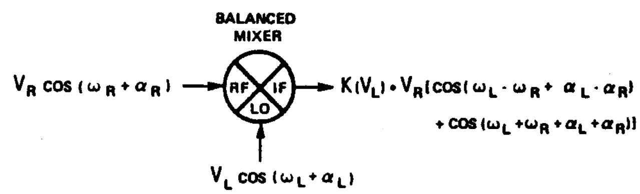

Fig. 2. Representation of a frequency mixer.

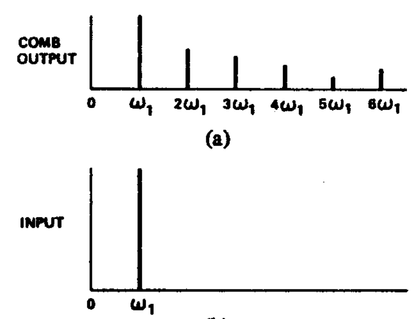

Fig. 3. Spectral display of comb generator input (b) and output (a).

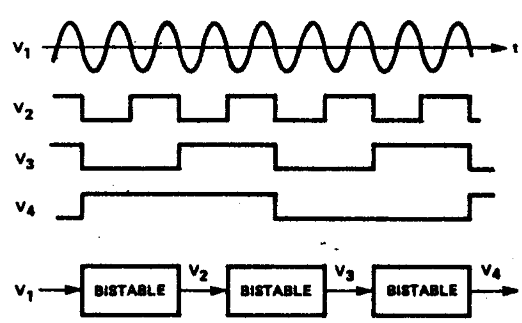

Fig. 4. Frequency divider and waveforms.

2) Multipliers: A frequency multiplier is used to produce a signal whose frequency is an integer multiple of the frequency of the driving signal. This is done by processing the driving signal through a nonlinear element. In general, if the driving signal is

$$V_1 \cos \theta_1 \quad (5)$$

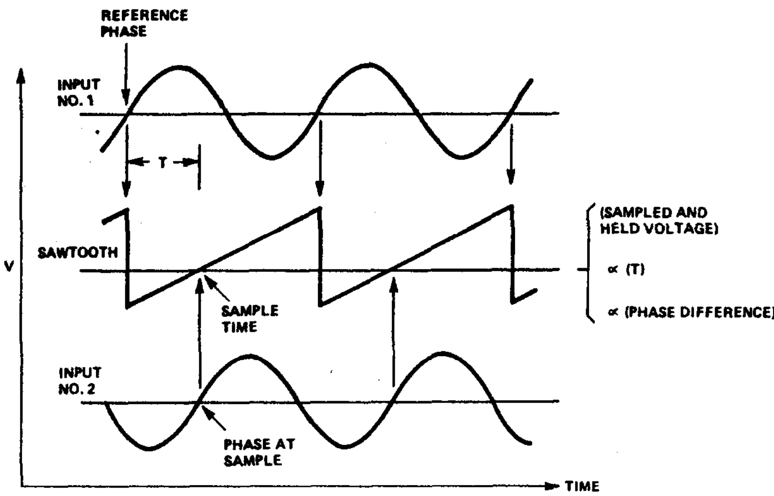

Fig. 5. Sample-and-hold phase detector waveforms.

where

$$\theta_1 = \omega_1 t + \alpha_1$$

the result of passing it through the nonlinearity will be

$$V_x = A_0(V_1) + A_1(V_1)\cos\theta_1 + A_2(V_1)\cos 2\theta_1 + \cdots + A_k(V_1)\cos k\theta_1 + \cdots. \quad (6)$$

$A_k(V_1)$ will be proportional to V_1^k at low enough drive levels but multipliers are often operated at high levels, where $A_k(V_1)$ approaches a constant. Multipliers that produce significant power at many multiples of the drive frequency are called comb generators because of the appearance of their output spectrums (Fig. 3). The driving power can be more efficiently transferred to a desired output frequency if tuning is used to concentrate the power at that one frequency.

A nonlinear element that is commonly used in frequency multipliers is the step-recovery diode. When a step-recovery diode is reverse biased, after it has been conducting in the forward direction, it continues to conduct for a while, but conduction ceases abruptly. This abrupt discontinuity in current flow can be used to generate the desired harmonics of the driving-signal frequency.

3) Dividers: Frequency dividers are nonlinear circuits that produce an output whose frequency is an integer submultiple of their input frequency. They are usually composed of bistable circuits, circuits which have one stable state that produces an output V_H and a second that produces an output V_L. In many realizations, the bistable changes state when its input passes some threshold between V_H and V_L while changing in a negative direction. In other bistables, the transition occurs when the input is moving in a positive direction. In either case, the output changes state only once for each input cycle so the output frequency is one-half the input frequency. Bistable elements can be cascaded and combined to produce various divider ratios. Fig. 4 illustrates a frequency divider composed of three bistable circuits.

4) Phase Detectors: A phase detector produces a voltage proportional to the difference in phase between two input signals. The balanced mixer is often used as a phase detector because, when the LO and RF signals are at the same frequency, its output contains a component of voltage that is proportional to their phase difference. This can be seen from (2) which, for $\omega_L = \omega_R$, is

$$V_2 = k_2 \{\tfrac{1}{2}V_L^2\,[1 + \cos 2(\omega_L + \alpha_L)] + V_L V_R \cos(\alpha_L - \alpha_R) + V_L V_R \cos(2\omega_L + \alpha_L + \alpha_R) + \tfrac{1}{2}V_R^2\,[1 + \cos 2(\omega_L + \alpha_R)]\}. \quad (7)$$

The second term in (7) gives a voltage proportional to the phase difference $\alpha_L - \alpha_R$. Other terms produced by the balanced mixer are partially removed by balancing or low-pass filtering. Because of difficulty in removing these undesired terms, a different type of phase detector, the sample-and-hold phase detector, is sometimes used.

In the sample-and-hold phase detector, a sawtooth waveform is synchronized to one of the input signals and its voltage is sampled and held on a capacitor each time the second signal reaches a given phase. The held voltage is then proportional, over a range of one cycle, to the difference in phase between the two input signals and, ideally, contains no other components. Fig. 5 illustrates this concept; often the two signals whose phases are being compared will be binary signals, rather than sinusoids.

II. The Method of Analysis

A. General Method

The method for analyzing the effects of small contaminating signals consists of decomposing the contaminating single-sideband (SSB) signal into AM and FM sidebands [1], [2] on the desired signal, treating the response to each set of sidebands separately, and recombining the separate results to obtain the total response. The desired input signal is assumed to be an unmodulated sine wave. The results will not be accurate if there is significant AM-to-PM or FM-to-AM conversion, but these effects are small in many practical situations. Further, in many practical situations the recombination will be simplified because the AM sidebands are suppressed sufficiently to be ignored at the output of the device.

We will associate the contaminants at the output with one of the output signals which would be produced in the absence of any input contaminants. However, nonlinear devices in general produce signals at many frequencies in response to a single-frequency input. Often the contaminants associated with these additional responses will be rejected by filters which are

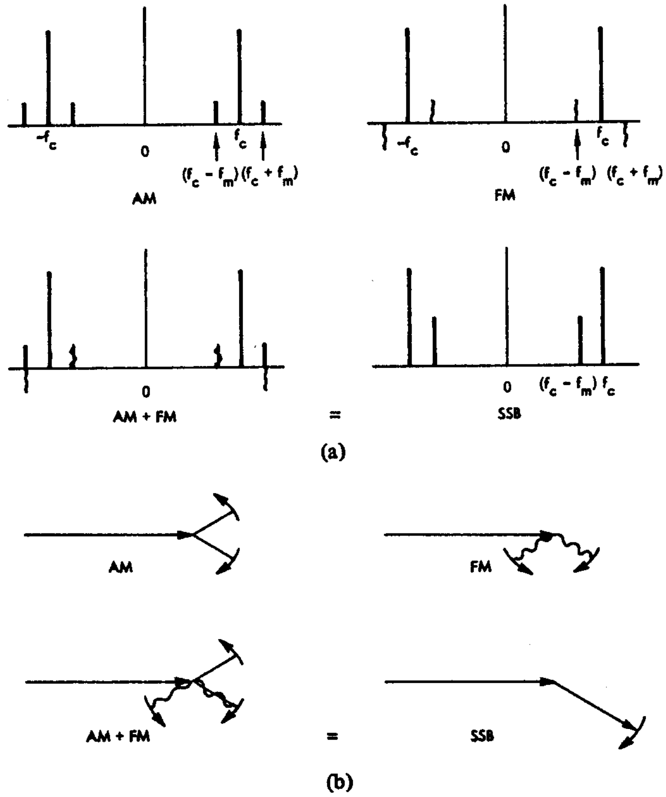

Fig. 6. An illustration of how an SSB can be composed of AM and FM sidebands, using (a) the Fourier representation and (b) the phasor representation (wiggly lines are used for FM sidebands for illustrative clarity).

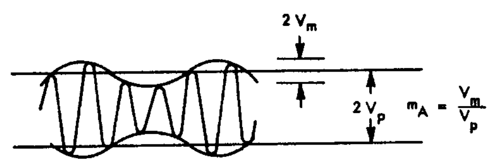

Fig. 7. AM voltage definitions for a sinusoidally modulated sine wave.

intended to reject the additional responses. However, we should be aware of the possibility that a contaminating signal associated with some additional response may become significant, perhaps by coming into the passband of a filter which passes the desired output.

The equivalence of SSB modulation to AM plus FM is illustrated in Fig. 6 in both the Fourier and phasor representations of the signals. Both representations of the FM signal lose accuracy as the sidebands become larger because the additional sidebands that are present with FM grow relative to the first sidebands, which are the only sidebands used in this approximation. (At a modulation index of 0.2, the first FM sidebands are 0.10 and the second sidebands are 0.005 relative to the carrier.) The SSB is, then, decomposed into a pair of AM sidebands and a pair of FM sidebands, all four of which are one half the amplitude of the original SSB.

The manner in which FM passes through the nonlinearities follows simple theoretical rules, which will be given later. There is a sampling effect, that occurs in frequency dividers, which could be considered somewhat complex, but which will also be shown to be predictable without reference to particular device characteristics.

AM is expected to be transferred according to the slope of the device gain curve at the operating point. This curve would normally be a plot of output power P_O versus power of the strong input signal P_I, both expressed in decibels, and for identical resistive impedance levels. The slope is

$$S = \frac{dP_O|_{\text{dB}}}{dP_I|_{\text{dB}}} = \frac{d \ln V_O}{d \ln V_I} = \frac{(1/V_O)\, dV_O}{(1/V_I)\, dV_I} \tag{8}$$

where V_O and V_I are the output and input signal amplitudes. Thus, the slope of this curve gives the ratio of the relative output voltage change (dV_O/V_O) to the relative input voltage change (dV_I/V_I) and, therefore, the ratio of the relative AM sideband level at the output to the relative AM sideband level at the input. In symbols, we have

$$m_{AO} = \frac{V_{mO}}{V_{pO}} = \frac{V_{mI}(dV_O/dV_I)}{V_{pI}V_O/V_I} = S\,\frac{V_{mI}}{V_{pI}} = S m_{AI} \tag{9}$$

where the variables are defined in Fig. 7 and the O and I subscripts represent output and input respectively. In many situations, the nonlinear device will operate in a saturated condition where S can be taken as zero.

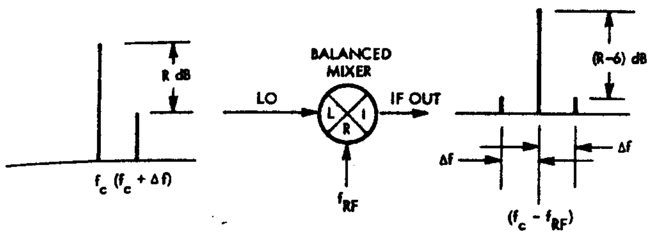

Fig. 8. Effect of a small SSB on the LO of a saturated (by LO) mixer.

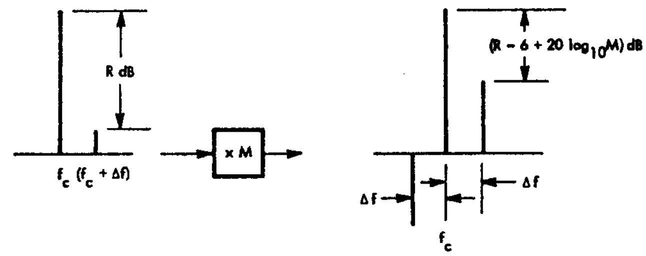

Fig. 9. Effect of a small SSB at the input of a saturated multiplier, ideal case.

If the nonlinear device is broad band, such that there is no significant FM-to-AM conversion, and if the AM-to-FM (or -PM) conversion is small enough, we can treat the AM and FM sidebands separately and simply recombine them at the output.

B. Frequency Mixers

In this paper, we will concentrate on a commonly used type of mixer, the balanced mixer, in which the strong LO signal level is reduced at the IF port by balanced circuitry. The mixer transforms the weaker (RF) input signal in an effectively linear manner, under usual operating conditions, during conversion to the IF. Mixers are often specified by the levels of spurious outputs produced at frequencies which are the sums and differences of multiples of the RF and LO frequencies and by intercept points, which describe the level of products produced by mixing between two RF signals (IM's). Contaminating signals that enter the LO port produce some products that are independent of the RF signal and are attenuated, both by conversion loss and mixer balance, before reaching the IF port. While even these reduced levels can often have serious consequences, these signals can often be confined to frequencies not in the IF band by filtering of the LO. The effect that we will consider here is one by which a contaminating signal, separated from the LO frequency by Δf, produces sidebands on desired IF signals that are also separated from them by Δf. Since Δf may be quite small (it may represent the separation from the center of the LO spectrum of a component of close-in noise), this contamination can be difficult to reduce by filtering.

The analysis applies also to a single-diode mixer, where small signals accompanying the LO are not differentiated from RF signals, except by frequency range.

Since any change in LO frequency will cause a similar change in the frequency of the IF (assuming the usual sum or difference frequency at the IF), FM sidebands are transmitted from LO to IF with no change in amplitude relative to the amplitude of the signal being modulated. The degree of AM transfer from the LO to the IF can be judged from the slope $-S$ of the conversion-loss curve, a curve of RF-to-IF signal attenuation versus LO power, since this also gives P_O versus P_I (as defined earlier) for a constant level of the RF signal. If the LO input is represented by

$$V_L = A_L\ [\sin \omega_L t + r \sin (\omega_L + \Delta\omega) t] \tag{10}$$

the resulting IF signal is

$$V_I \approx A_I \left\{ \sin \omega_I t + \frac{r}{2}\ [\sin (\omega_I + \Delta\omega) t - \sin (\omega_I - \Delta\omega) t + S\ (\sin (\omega_I + \Delta\omega) t + \sin (\omega_I - \Delta\omega) t)] \right\}. \tag{11}$$

Here we have decomposed the single sideband into AM and FM sidebands and translated their frequencies along with the frequency of the desired signal while leaving the relative magnitude of the FM sidebands unchanged but multiplying the relative amplitude of the AM sidebands by S. This result is similar to one obtained by N. Blachman through a different derivation [3].

For low loss and good intermodulation suppression, the LO level is usually high enough so that S is much less than unity and the IF spectrum consists primarily of a carrier and two FM sidebands as shown in Fig. 8.

C. Frequency Multipliers

A saturated frequency multiplier will suppress AM sidebands in the same manner that a saturated mixer suppresses AM on its LO. Even if the multiplier is operated at a point where S exceeds unity, FM is likely to predominate because FM sidebands increase in relative amplitude by the multiplication factor M, as shown in Fig. 9. This is true because, when the input frequency f_I deviates by Δf, the output at Mf_I deviates by $M\Delta f$, but the modulation frequency f_m is the same at input and output. Thus the output FM index m_{FO} is given in terms of the input modulation index by

$$m_{FO} = \frac{M\Delta f}{f_m} = M m_{FI}. \tag{12}$$

While AM itself may be suppressed, the effects of AM-to-PM conversion may be severe in multipliers [4]. Where the harmonics are generated by a rapid transition synchronized to one slope of the input waveform (for example, with a step-recovery diode), as the amplitude of the input varies the phase at which the rapid transition occurs will change, causing AM-to-PM conversion. The FM so generated will be multiplied like any other FM at the input and will thus perturb the ideal results, even for high values of M.

In the absence of AM-to-PM and FM-to-AM conversion, prediction of the level of contaminating signals at the multiplier output involves:

1) resolution of the SSB into AM and FM sidebands, each 6 dB smaller;
2) a gain of M in relative level for FM sidebands, due to multiplication by M;
3) multiplication of AM sidebands by the AM transfer gain S_M;
4) recombination of the resulting AM and FM sidebands, adding sidebands that are on the same side of the output carrier as the original SSB was, relative to the input signal, and subtracting sidebands on the other side (assuming $S_M > 0$).

If the input to the multiplier is represented by (10), this process gives a multiplier output of

$$V_M \approx A_M \left\{ \sin M\omega_L + \frac{r}{2} [M \langle \sin (M\omega_L + \Delta\omega)t - \sin (M\omega_L - \Delta\omega)t \rangle + S_M \langle \sin (M\omega_L + \Delta\omega)t + \sin (M\omega_L - \Delta\omega)t \rangle] \right\} \tag{13}$$

where S_M is the slope of the logarithmic curve of Mth-harmonic power versus input power, as in (8).

An SSB is decomposed into two in-phase and two 180°-out-of-phase (with each other) components, as shown in Fig. 6. If the AM and FM sidebands so represented are multiplied by different constants, each with the same phase, they will then add in such a manner as to produce a larger sideband on the side where the SSB originally existed. However, in the case of AM-to-PM conversion, we expect the resulting phase deviation to be in phase with the amplitude deviation that caused it, but Fig. 6 represents amplitude and phase deviations that are in quadrature. Therefore, the PM sidebands produced due to AM-to-PM conversion tend to be in quadrature with all of the components of the original decomposition and cannot cancel any of them.

D. Frequency Dividers

The most commonly used frequency dividers are regenerative circuits which change state when the input (clock) moves through a relatively narrow part (the transition region) of its total normal voltage swing. The signal level at the divider output is largely independent of the level at the input, so significant AM suppression is to be expected. If the input is so biased as to be centered on the transition region, AM-to-PM conversion should also be minimum. If the bias is shifted from this point, however, a change in signal amplitude will cause a change in the time at which this transition region is passed and, thus, AM-to-PM conversion. The bias that gives the best sensitivity (i.e., permits dividing with the smallest input signal) is approximately the same as the bias which minimizes AM-to-PM conversion, since the signal would be centered in the transition region in each case.

If AM is suppressed completely, the output will have only FM sidebands whose level relative to the carrier is lower by the divide ratio N than the relative level of the input FM sidebands, as illustrated in Fig. 10. The reason for reduction by a factor N is basically the same as the reason that the multiplier sidebands increase by M.

1) AM-to-PM Conversion: If we assume that the transition occurs when the input passes a certain level (the transition level), then the phase at the transition will be

$$\theta_T = \sin^{-1} \frac{V_T}{V_p} \tag{14}$$

where the variables are defined in Fig. 11. The rate of change of phase with amplitude is

$$\frac{d\theta_T}{dV_p} = - \frac{1}{V_p \sqrt{\left(\frac{V_p}{V_T}\right)^2 - 1}}. \tag{15}$$

Therefore, the FM index m_F, which equals the peak phase deviation in radians, is related to the bias offset V_T, by

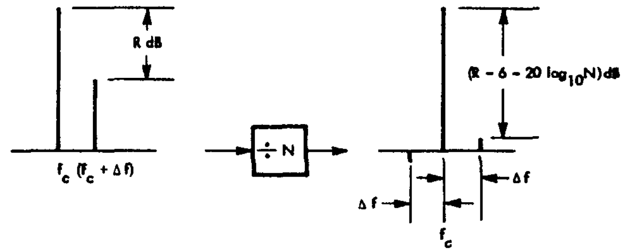

Fig. 10. Effect of a small SSB on the input to a divider biased for high sensitivity

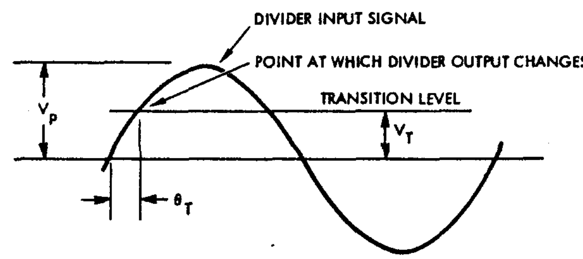

Fig. 11. Divider input signal and transition level.

$$m_F = \left| \frac{d\theta_T}{dV_p} \right| V_p m_A \tag{16}$$

$$= \frac{m_A}{\sqrt{\left(\frac{V_p}{V_T}\right)^2 - 1}} \tag{17}$$

and the effective FM sidebands at the input will be lower by

$$A = 20 \log_{10} \sqrt{\left(\frac{V_p}{V_T}\right)^2 - 1} \text{ dB} \tag{18}$$

than the AM sidebands which cause them. When an SSB at the divider input is decomposed into AM and FM, the phase modulation due to the FM will exceed that due to the AM if V_T is less than 70 percent of V_p. Therefore, the AM-to-PM conversion can often be ignored in dividers that are biased for good sensitivity relative to the input signal strength. When the bias offset is small, (18) becomes

$$A|_{V_T \ll V_p} \approx 20 \log_{10} \frac{V_p}{V_T} \text{ dB}. \tag{19}$$

At the divider output, the relative (to the carrier) amplitude of the FM sidebands due to AM at the input is

$$R_O \text{ dB} = R_I \text{ dB} - A - (20 \log_{10} N) \text{ dB} \tag{20}$$

where R_I dB is the relative (to the carrier) amplitude of the AM sidebands at the input.

2) Sampling: The output of the digital frequency divider supplies information concerning its phase only at the time of the discrete output transitions. Phase modulation at the divider input affects the output only at these times. Therefore, the phase of the divider output is sampled at the transition rate

$$f_s' = 2f_I/N \tag{21}$$

where f_I is the divider input frequency. If the divider drives a device that makes use of only one direction of transition, the

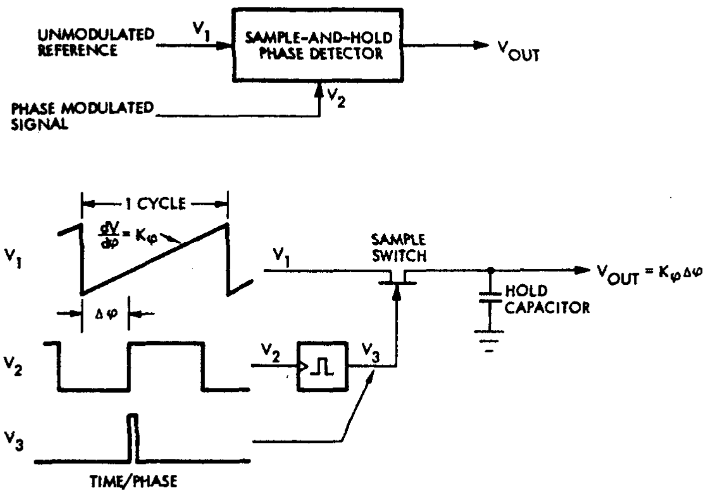

Fig. 12. Concept of the sample-and-hold phase detector. The linear part of V_1 represents changing phase. The sample switch connects V_1 to the hold capacitor at the upward transition of V_2.

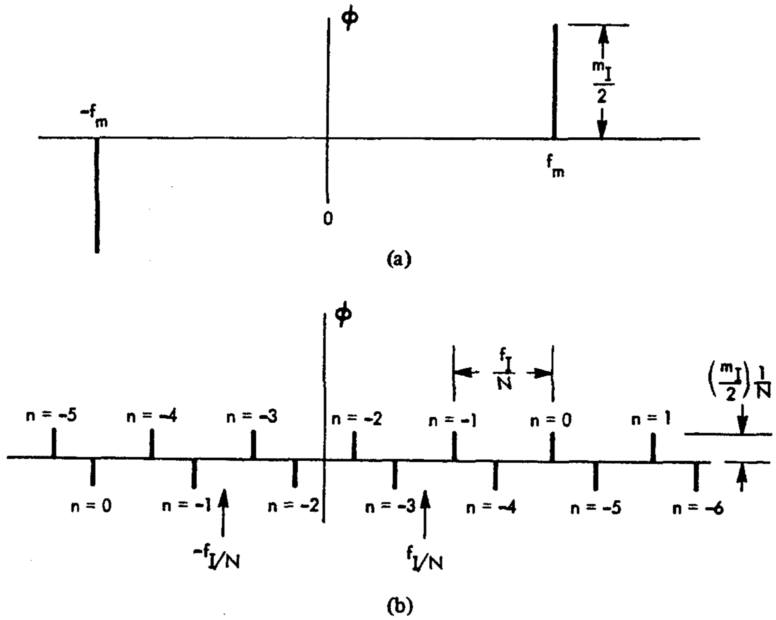

Fig. 13. Fourier spectrum of phase deviation (a) at divider input and (b) for positive transitions at output, before data hold.

sampling rate will be half of (21), or

$$f_s = f_I/N. \tag{22}$$

For example, assume that the divider input is phase modulated with a peak deviation m_I at the frequency f_m, and its output drives a sample-and-hold phase detector that samples on its upward transitions, as illustrated in Fig. 12. The phase-detector output will contain not only a component at f_m, but components at all frequencies given by

$$f_d = |f_m + nf_s| \tag{23}$$

$$= |f_m + nf_I/N| \tag{24}$$

where n is any positive or negative integer [5]. (Although the input is modulated, we approximate f_I as a constant in (24)). This is illustrated in Fig. 13. The amplitudes of all of these components will be identical except for multiplication by the zeroth-order-hold transfer gain [6],

$$|H| = \left|\frac{\sin \pi\,(f_d/f_s)}{\pi\,(f_d/f_s)}\right|. \tag{25}$$

In terms of the input modulation, each output component will have a magnitude given by

$$V = \frac{K_\varphi m_I}{N} \frac{\sin \pi\,(Nf_d/f_I)}{\pi\,(Nf_d/f_I)} \tag{26}$$

where K_φ is the ratio of output voltage to input phase for the phase detector. This means that an input phase modulation at a frequency given by

$$f_m = (K + \delta)\frac{f_I}{N} \tag{27}$$

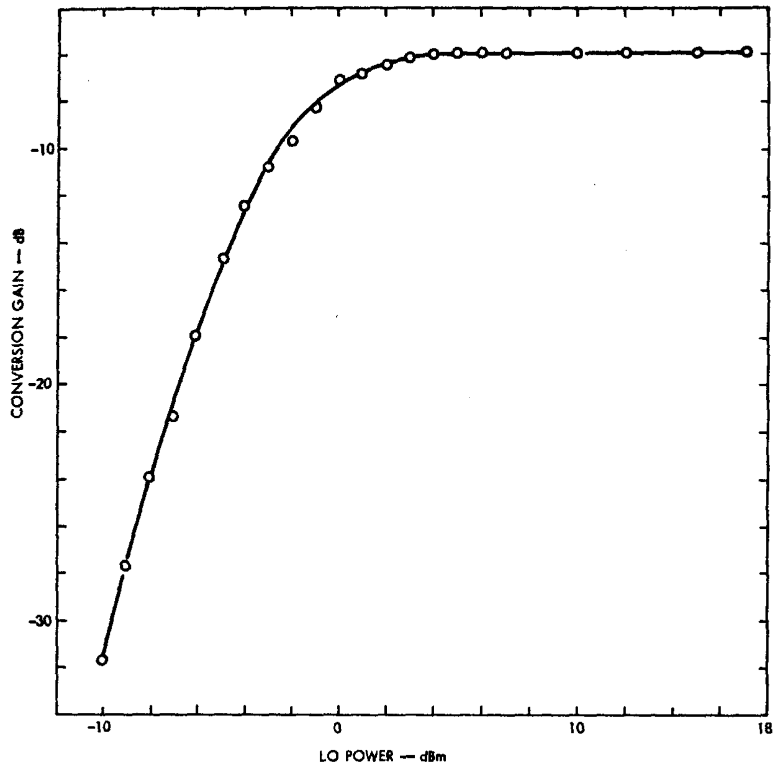

Fig. 14. Conversion gain versus LO power.

where K is any integer and δ is small compared to one, will appear at the phase-detector output with a frequency

$$f_d = \left| \delta \frac{f_I}{N} \right|. \tag{28}$$

This can be shown by combining (24) and (27). The amplitude of this component at the phase-detector output is given by (26) as

$$V = \frac{K_\varphi m_I}{N} \frac{\sin \pi\delta}{\pi\delta} \approx \frac{K_\varphi m_I}{N}. \tag{29}$$

Thus there are an infinite number of input modulation frequencies that will produce this low-frequency output, and the attenuation at each will be the same. For example, if the divider input frequency is 1 MHz and its output is at 1 kHz, an FM sideband on the divider input, at 1100.1 kHz, will produce the same output from the phase detector as would an FM sideband of identical level at 1000.1 kHz. This is of special importance in phase-locked synthesizers because contaminating signals that are separated from the desired signal by a frequency much greater than the loop bandwidth can cause phase modulation well within that bandwidth, and this modulation will be followed by the output signal.

While we have used a sample-and-hold phase detector in this example, the sampling process is inherent in the operation of the divider, and similar translations are to be expected when other devices are driven by the divider.

3) Summary: The process for analyzing the effects of an SSB contaminating signal at a divider input is summarized as follows:

First: Decompose the SSB into AM and FM sidebands.

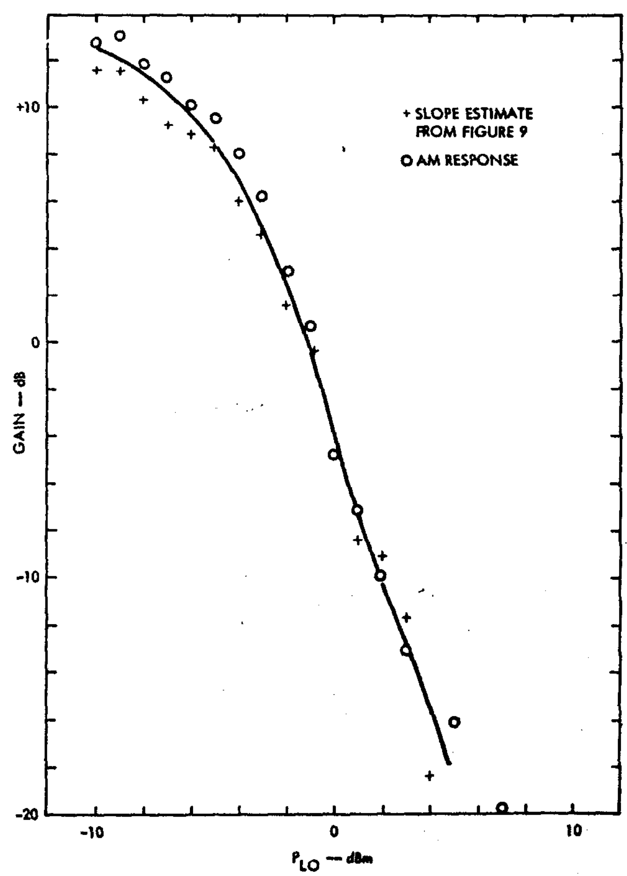

Fig. 15. AM transfer gain.

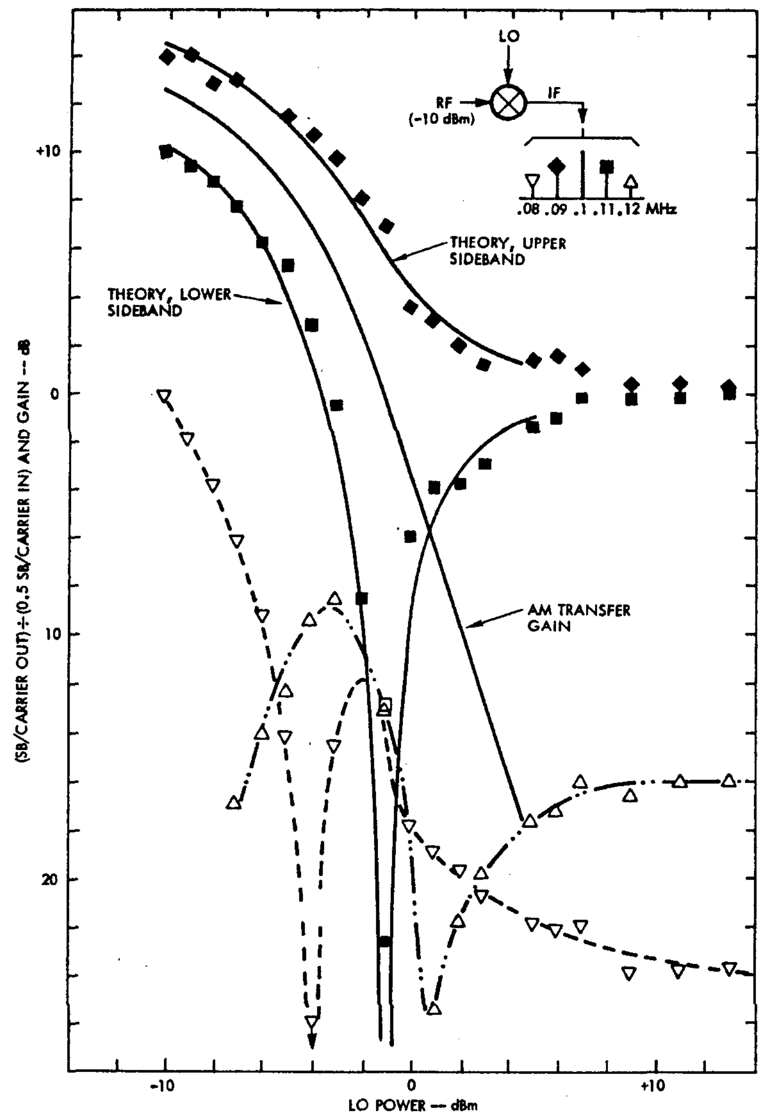

Fig. 16. Theoretical and measured mixer output sideband levels.

Second: Determine or estimate the amount of AM-to-PM conversion.

Third: Determine the total FM sidebands at the input by adding, in quadrature, the FM sidebands obtained in the first step to the FM sidebands determined in the second step.

Fourth: Reduce the sideband levels by the divide ratio N.

Fifth: Apply (23) to find all of the modulation frequencies generated.

III. Experimental Results

A. Frequency Mixing

Measurements were made employing a double-balanced mixer (Relcom M1A) with a signal level of -10 dBm. The conversion gain is shown versus LO power in Fig. 14. In order to determine the degree of AM transfer from LO to IF, the LO was amplitude modulated 20 percent and the resulting AM sidebands were measured at the IF. Fig. 15 shows the AM transfer ain from LO to IF with points plotted from the AM measurements and from estimates of the slopes of the curve in Fig. 14. The two sets of data points show good agreement. AM was then stopped and an upper-sideband contaminating signal was added to the LO at a level 14 dB below the LO power. By Section II-A, this is equivalent to two AM and two FM sidebands, each at -20 dB. The sideband-to-carrier ratios observed at the IF port are plotted in Fig. 16 relative to the -20-dB level that would be obtained from FM alone if AM were completely suppressed. Also shown is the AM transfer gain from Fig. 15 and the theoretically expected levels of the two main sidebands from (11). The measurements show good agreement with theory. The levels of the second-order FM sidebands are also plotted, and it can be seen that they are relatively small. Table I lists the frequencies employed in this experiment.

TABLE I
Frequencies Used in the Mixer Test

LO (desired signal)	10 MHz
AM on LO	10 kHz
Contaminating Signal with LO	10.01 MHz
RF (Weak) Signal	10.1 MHz
IF	0.1 MHz

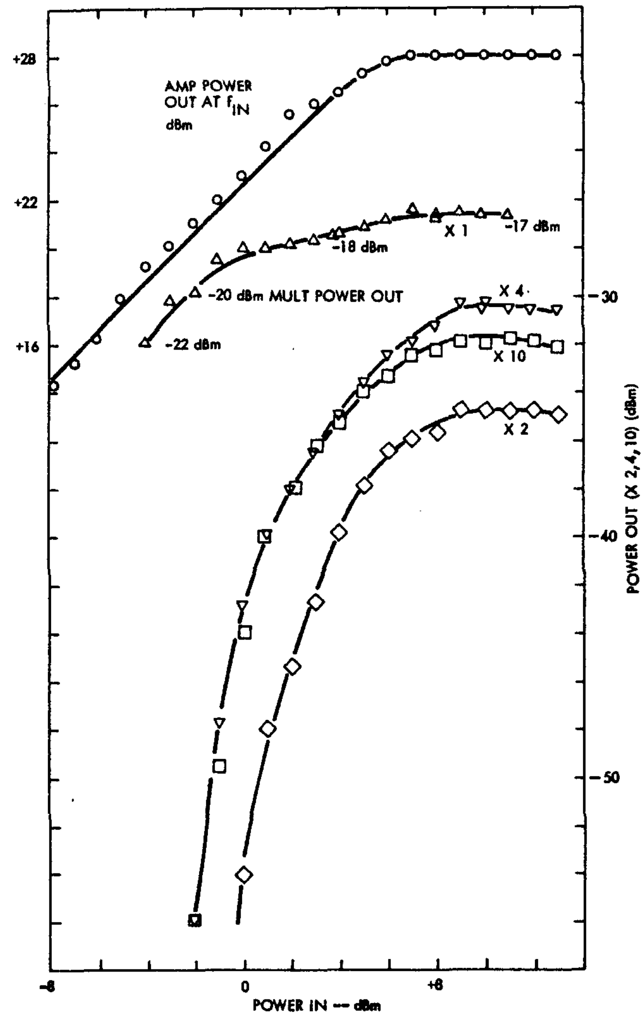

Fig. 17. Output power from multiplier and driving amplifier versus input power.

B. Frequency Multiplication

Measurements were made on a frequency multiplier consisting of a spectrum generator (HP H21-33002A) driven through an amplifier (Avantek AWP400T) at 100 MHz.

Fig. 17 shows power output versus input (the power transfer curve). Fig. 18 shows the expected sideband level at the doubled output, as estimated from the slope S_2 of the X2 curve in Fig. 17, and also the measured AM transfer gain to the doubled output. At higher power levels, the measured AM transfer gain is higher than Fig. 17 would indicate, suggesting significant AM-to-PM conversion. The difference in levels of the two output sidebands in the region above 7-dBm input power, where the amplifier has saturated, is explainable by AM-to-PM conversion if there is some phase shift between the AM and resulting PM.

Fig. 18 also confirms the theoretical 6 dB increase in FM sideband levels through the doubler. The measured sideband levels on the doubled output, in response to a - 24-dBm SSB, are shown in Fig. 19, where they are compared to values predicted by (13) with S_2 obtained from the measured AM transfer gain from Fig. 18 (SSB offset equals AM frequency). In Fig. 20 these same sideband levels are compared to levels predicted by (13) with S_2 obtained from the slope of the power transfer curve, as in Fig. 18. Both methods give useful results except where the predicted level is very small. The actual lower sideband was not observed to disappear; its minimum level corresponds to a 14° (+180°) phase difference between the AM and FM sidebands.

Tables II and III show the parameters involved in prediction of the transfer of a contaminating signal through the X 4 and X 10 multiplication process, respectively, for three power levels in each case. Since PM is multiplied, AM-to-PM conversion is accentuated for higher order multipliers, as can be seen through a comparison of columns (b) and (c). Columns (f) give predictions based on the slope of the power transfer curves. Columns (g) give predictions based on the measured AM transfer gain. Consideration of the FM component alone suffices for X 2, X 4 and X 10 when the multiplier is well saturated. These predicted levels are a - 24-dB sideband for X 2 (compare to Fig. 18) and each sideband 6 dB and 14 dB higher than the SSB for X 4 and X 10, respectively (compare to columns (e) in Tables II and III). Where the power transfer curve has near-unity slope or where the slope is steep, neither prediction method gives consistent accuracy for lower sidebands, but all predictions of upper sideband levels are within a few decibels of measured values for upper sidebands and come close to being an upper bound on the magnitude of the lower sideband.

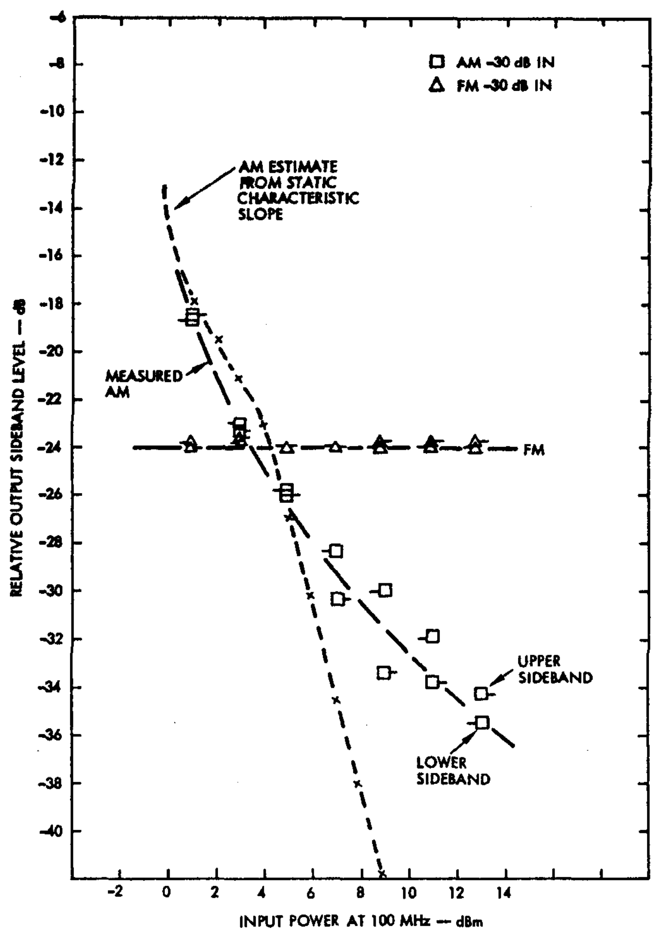

Fig. 18. AM and FM transfer through doubler.

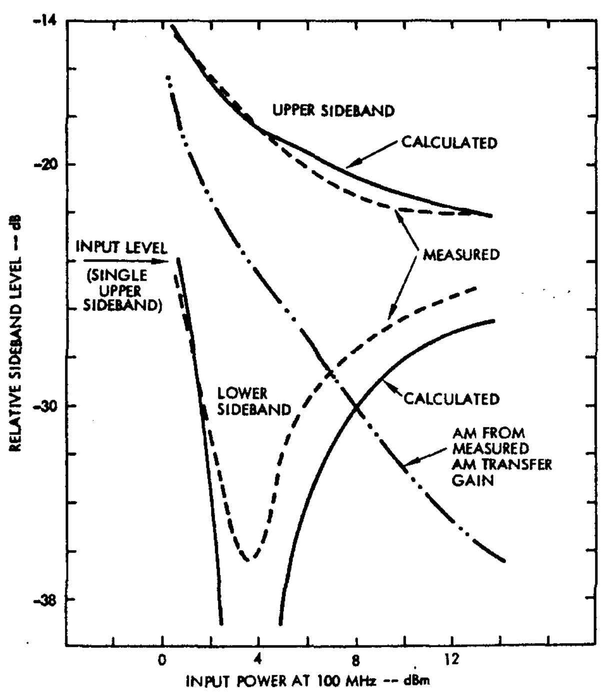

Fig. 19. Measured response of doubler to SSB plus prediction based on AM measurements.

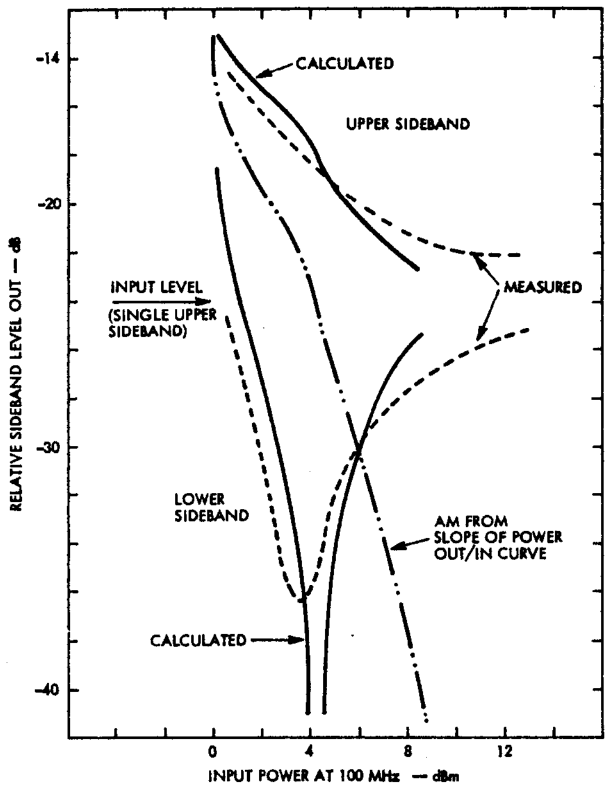

Fig. 20. Measured response of doubler to SSB plus prediction based on power transfer curve.

TABLE II
PARAMETERS RELATING TO THE TRANSFER OF CONTAMINATING SIGNALS TO THE ×4 OUTPUT. VALUES ARE GIVEN FOR BOTH SIDEBANDS

(a)	(b)	(c)	(d)	(e)	(f)	(g)
P_{Gen} (dBm)	S, Slope (dB)	Measured AM Gain (dB)	Measured FM Gain (dB)	Measured** SSB Gain (dB)	Theory, SSB (dB) from** (b)	(c)*
+13	→ − ∞	2.1/2.1	+12.3/+12.3	+6.1/+7.6	6/6	4.1/7.6
+6	-0.7 dB	8.1/7.5	+12.1/+12.2	+7.8/+8.2	3.7/7.8	5.1/6.8
-1	+16.3 dB	18.1/19.2	+12.2/+12.2	+9.9/+16	2.1/14.4	7.1/16.0
THEORY:	b=c		+12		e=f	e=g

*Using average of 2 values.
**Each SB out/SSB in.

TABLE III
PARAMETERS RELATING TO THE TRANSFER OF CONTAMINATING SIGNALS TO THE ×10 OUTPUT. VALUES ARE GIVEN FOR BOTH SIDEBANDS

(a)	(b)	(c)	(d)	(e)	(f)	(g)
P_{Gen} (dBm)	S, Slope (dB)	Measured AM Gain (dB)	Measured FM Gain (dB)	Measured** SSB Gain (dB)	Theory, SSB (dB) from** (b)	(c)*
+13	→ − ∞	10.2/10.3	20.2/20.2	14.3/14.2	14/14	10.5/16.5
+4	-2.9 dB	18/18	20.7/20.7	15.7/16.2	13.4/14.6	0.3/19.1
+1	+6.4 dB	22.2/22.2	20.1/20.1	17.5/19.2	12.0/15.6	3.2/21.2
THEORY:	b=c		+20		e=f	e=g

*Using average of 2 values.
**Each SB out/SSB in.

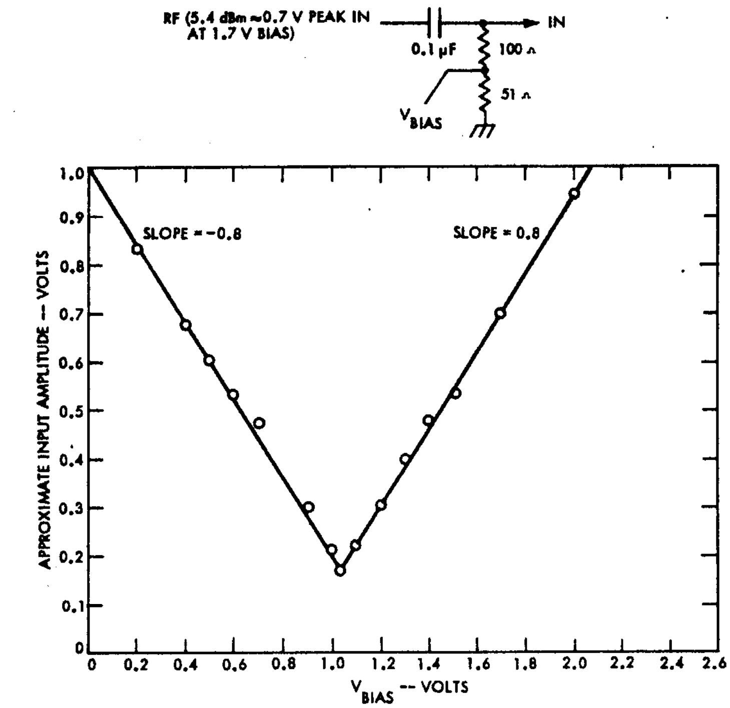

Fig. 21. Required amplitude into a 54LS197 (÷16) versus bias at 10.24 MHz.

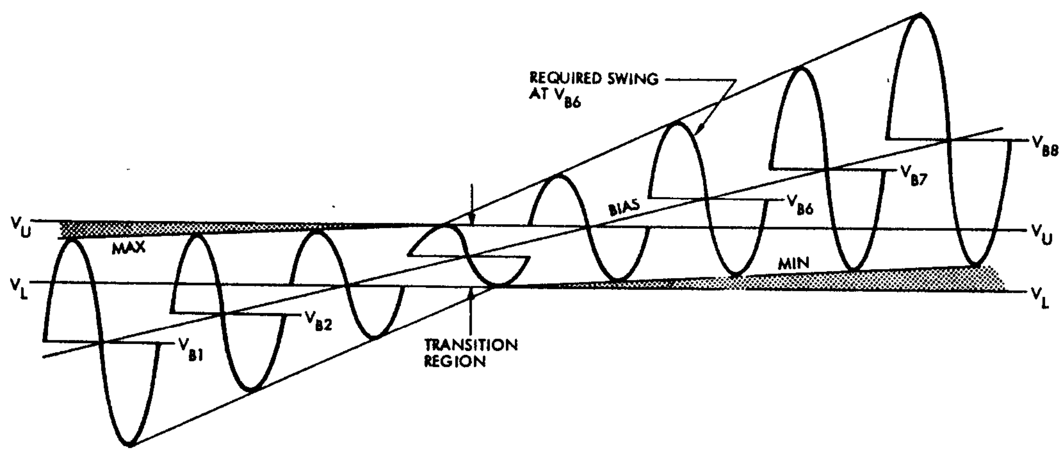

Fig. 22. Change in required input swing with bias. The swing must pass through a transition region whose boundaries change slightly with input amplitude.

C. Frequency Division

Measurements were made on a standard 54LS197 frequency divider (÷16) IC [7]. At various input bias levels, the input signal was reduced until the divider failed to divide properly. The required input amplitude is plotted versus bias in Fig. 21. The bias voltage is the externally supplied voltage; the amplitude was obtained by measuring the peak-to-peak swing, for one source setting, at the divider input with the bias at a level where no clipping was apparent, and multiplying by the change in (50-Ω) attenuator setting from there. Other measures are possible, but this method has the advantage of experimental ease. Fig. 21 shows a minimum required amplitude of 0.16 V at a bias of 1.04 V. A change in bias from the optimum must be compensated for by increasing amplitude by only 80 percent of that change. Thus, when the signal becomes larger, its excursion need not reach quite to the same extreme as for a smaller signal. This is illustrated by Fig. 22.

The divider input was amplitude modulated 20 percent and the resulting sidebands on the ÷2 and ÷16 outputs are shown in Fig. 23, as functions of the bias, along with some theoretical curves. Curve *A* is a plot of (17), reduced by the divide ratio, and shows that the equation gives a fair match to experimental data. A better match is obtained by using, in (17), an effective signal amplitude that is larger by 1/0.8 to account for the apparent change in threshold with bias as shown in Figs. 21 and 22; such curves are shown as *B* in Fig. 23. The measured values deviate from curve *B* at low output-sideband levels and at low bias levels. The former discrepancy may be due to the coupling of AM to the output. The latter is probably due to suppression of AM at the input due to clipping when the swing goes

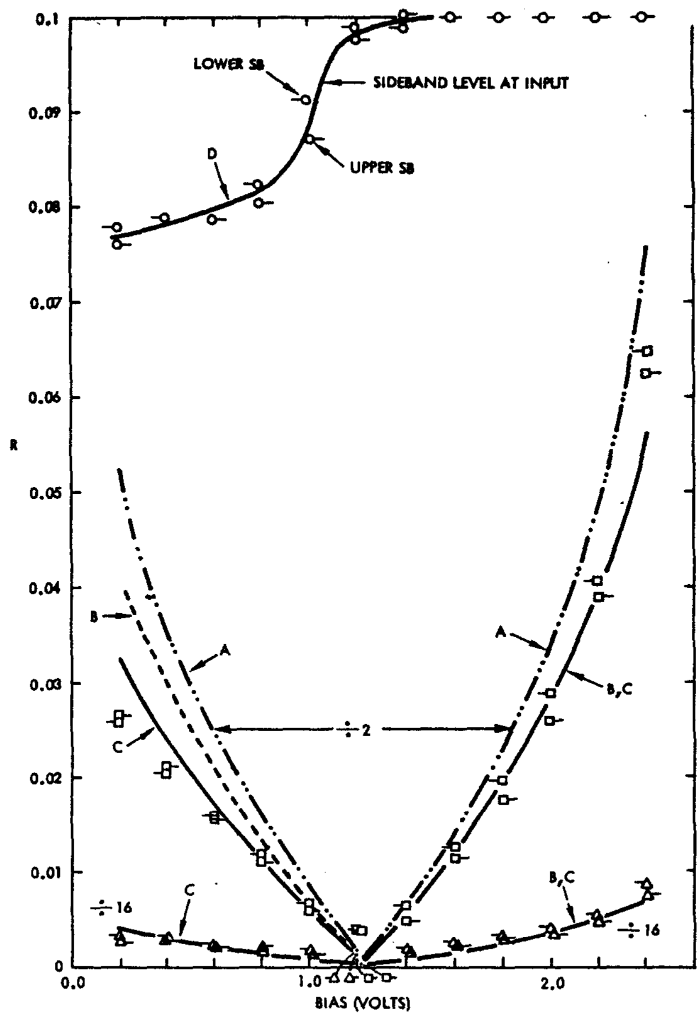

Fig. 23. Sideband levels at ÷2 and ÷16 outputs due to AM on 2-V peak input. Curves *A–C* are from (17), but *B* and *C* use an effective input amplitude increased by 1.25 and *C* is also based on the input AM level indicated by curve *D*.

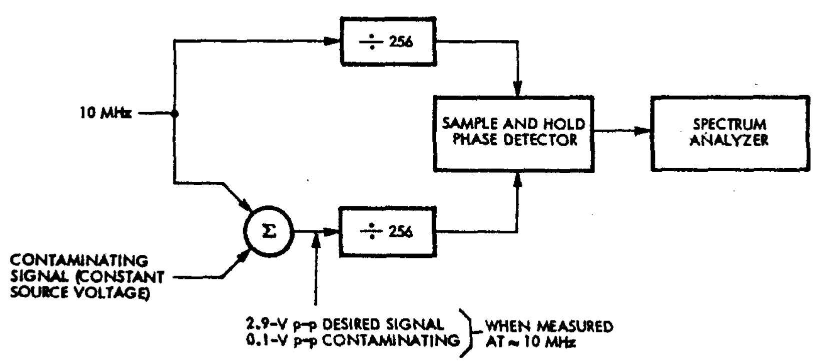

Fig. 24. Test setup for showing sampling effect in dividers.

too far negative; this AM suppression at the input is shown by curve *D* and an improved match is obtained by using this reduced modulation level, as is done in curves *C*.

In order to observe the effect of sampling, two dividers were driven by a 10-MHz signal and, in addition, one divider had an added contaminating signal at its input. The setup is shown in Fig. 24. The dividers were biased near maximum sensitivity and the phase-detector output was predicted under the assumption of no AM-to-PM conversion. The contaminating-signal frequency was offset from the desired-signal frequency by 2 kHz and the measured output phase deviation was found to be within a few tenths of a decibel of theoretical. The contaminating-signal frequency was then shifted from ±2 kHz by multiples of the divider's output frequency and the 2-kHz component on

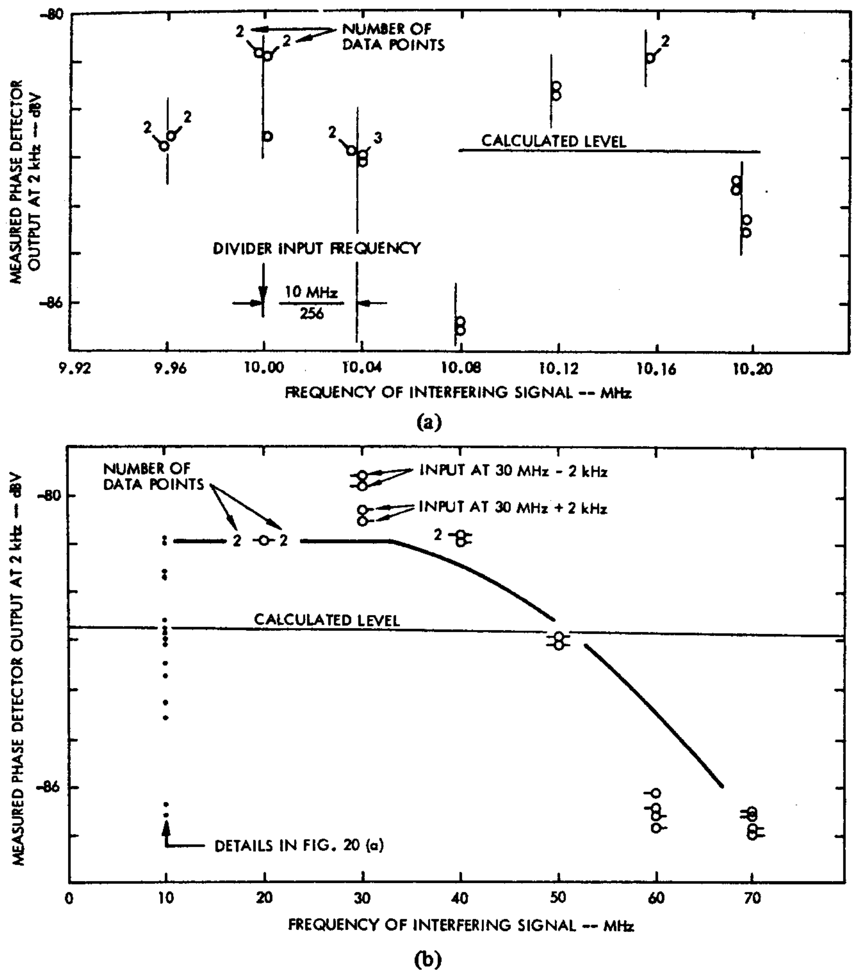

Fig. 25. Phase detector output at 2 kHz resulting from frequency division of contaminated signal with contamination at various frequencies.

the phase-detector output was again measured. Results are shown in Figs. 25(a) and (b). The resulting phase deviation is within about 4 dB of the theoretical constant level, even when the contaminating-signal frequency is as high as 70 MHz, which is well above the typical maximum toggle frequency, 40 MHz, for this type of divider [7].

IV. Conclusions

We have seen how the effects of small contaminating signals in mixers, multipliers and dividers can be predicted by decomposition of the contaminating signal into AM and FM sidebands, which are then treated separately. This method is simplest to apply and most accurate under the most common operating conditions of the devices. Results are poorest where there is significant AM-to-PM and FM-to-AM conversion or when the effects of the contaminating signals are greatly attenuated, relative to the desired output, by the nonlinear device.

Contaminating signals (or other phase modulation) in frequency dividers produce many modulation frequencies at the divider output due to frequency translation caused by a sampling effect.

Acknowledgment

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