Rate Decimation Filters for SoftWave

Rate Decimation Filtering for SoftWave Project
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Reference: "Digital T/2 Nyquist Filtering Using Recursive All-Pass Two-State Resampling Filters for a Wide Range of Selectable Signaling Rates," 1992, Fred Harris & Itzhak Gurantz

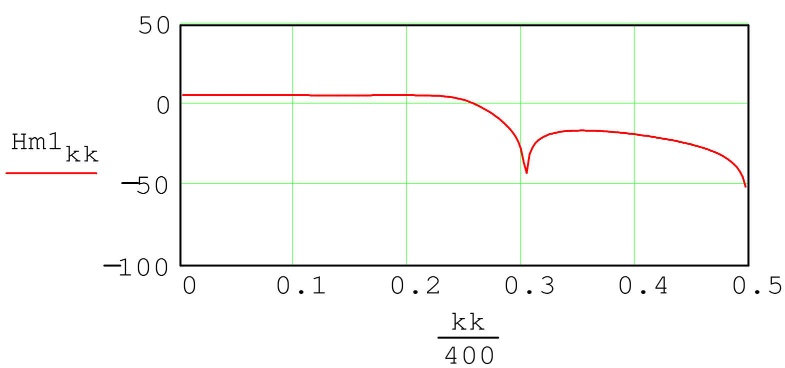
$kk := 1 .. 199$

$z_{kk} := \exp\left(\sqrt{-1}\cdot 2\cdot\pi\cdot\frac{kk}{400}\right)$ $\qquad$ $delw := \frac{2\cdot\pi}{400}$

$\alpha 0 := 0.11$ $\qquad$ $\alpha 1 := 0.65$

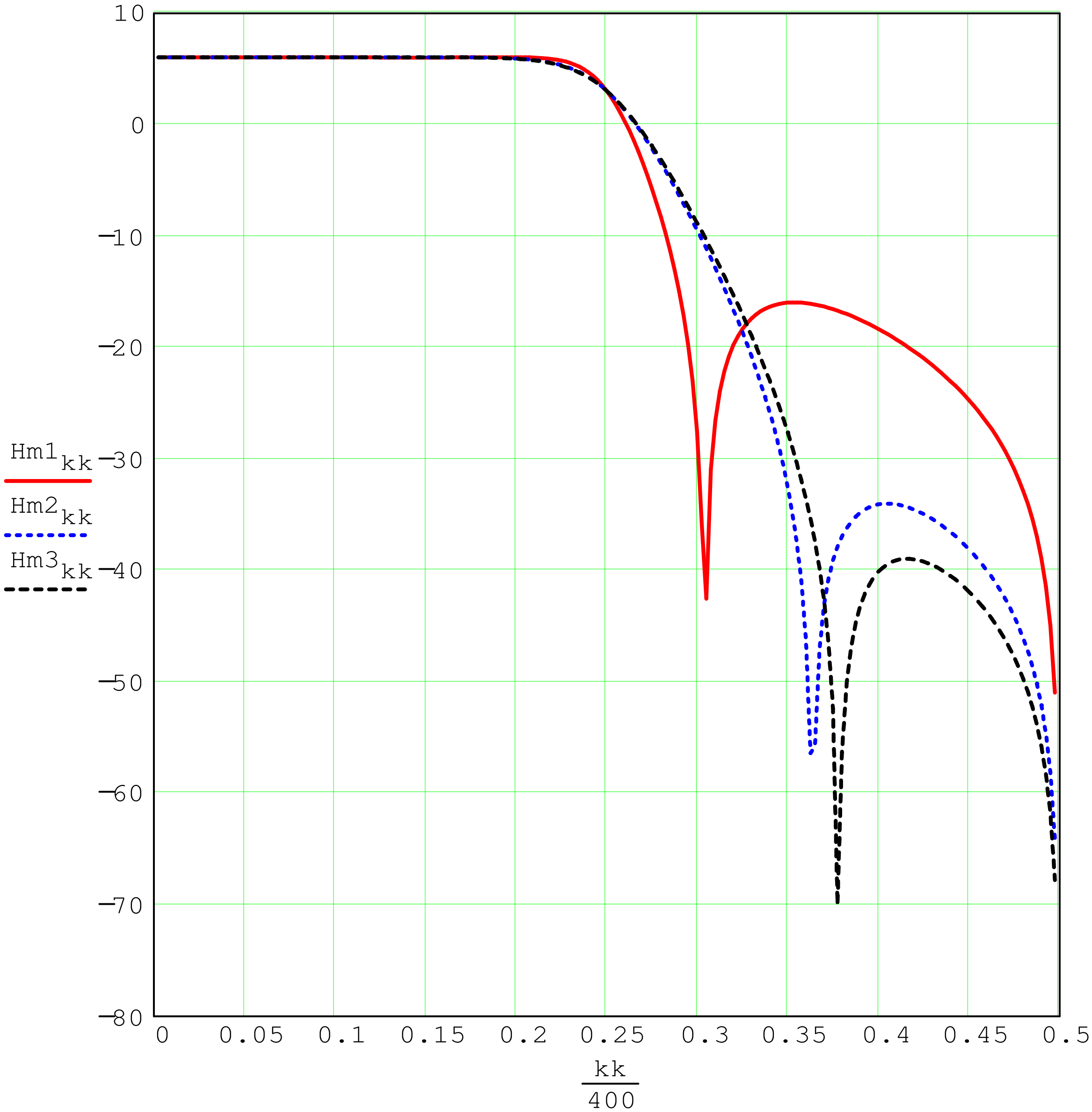
$$H(s, \alpha 0, \alpha 1) := \left(\frac{1+\alpha 0\cdot s^2}{s^2+\alpha 0}\right) + \frac{1}{s}\cdot\frac{1+\alpha 1\cdot s^2}{s^2+\alpha 1}$$

$$Hm1_{kk} := 10\cdot\log\left[\left(\left|H(z_{kk}, \alpha 0, \alpha 1)\right|\right)^2\right]$$



$$Hm2_{kk} := 10\cdot\log\left[\left(\left|H(z_{kk}, 0.105, 0.55)\right|\right)^2\right]$$

$$Hm3_{kk} := 10\cdot\log\left[\left(\left|H(z_{kk}, 0.104, 0.54)\right|\right)^2\right]$$


10
0
−10
−20
−30
−40
−50
−60
−70
−80
Hm1kk
Hm2kk
Hm3kk
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
kk
400

$jx := 2 \cdot \pi \cdot \sqrt{-1}$

$$H2(s, a0, a1, a2, a3) := \left(\frac{1 + a0 \cdot s^2}{s^2 + a0}\right) \cdot \left(\frac{1 + a1 \cdot s^2}{s^2 + a1}\right) + \frac{1}{s} \cdot \left(\frac{1 + a2 \cdot s^2}{s^2 + a2}\right) \cdot \left(\frac{1 + a3 \cdot s^2}{s^2 + a3}\right)$$

$a0 := 0.1 \quad a1 := 0.5$

$a2 := 0.6 \quad a3 := 0.5$

$b_1 := 0 \quad b_0 := 0 \quad b_2 := 0 \quad b_3 := 0$

Given

$$10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.1}, a0, a1, a2, a3\right)\right|\right)^2\right] = 6.0$$

$$10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.25}, a0, a1, a2, a3\right)\right|\right)^2\right] = 6.0$$

$$\left[10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.30}, a0, a1, a2, a3\right)\right|\right)^2\right] + 50\right] \cdot 10 = 0$$

$$\left[10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.325}, a0, a1, a2, a3\right)\right|\right)^2\right] + 60\right] \cdot 10 = 0$$

$$\left[10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.35}, a0, a1, a2, a3\right)\right|\right)^2\right] + 66\right] \cdot 10 = 0$$

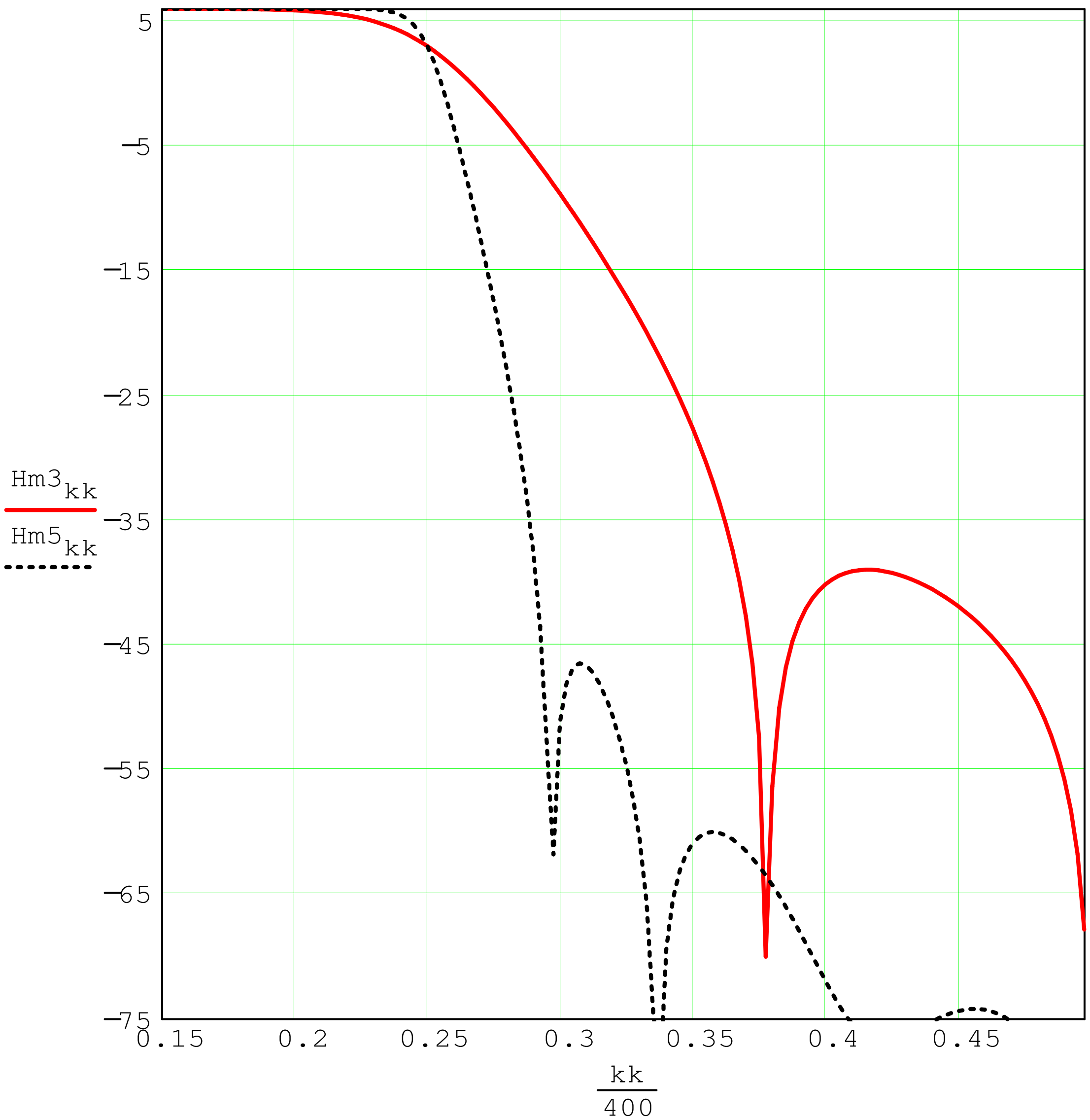
$$\left[10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.40}, a0, a1, a2, a3\right)\right|\right)^2\right] + 70\right] \cdot 10 = 0$$

$$\left[10 \cdot \log\left[\left(\left|H2\left(e^{jx \cdot 0.45}, a0, a1, a2, a3\right)\right|\right)^2\right] + 75\right] \cdot 15 = 0$$

$b := \text{minerr}(a0, a1, a2, a3)$

$$b = \begin{pmatrix} 0.065019 \\ 0.511626 \\ 0.822046 \\ 0.246975 \end{pmatrix}$$

$$Hm5_{kk} := 10 \cdot \log\left[\left(\left|H2\left(z_{kk}, b_0, b_1, b_2, b_3\right)\right|\right)^2\right]$$



Using the Noble Identity, the 7th order filter can be implemented by alternately evaluating one of the two path filters on each new input sample rather than a complete 7th order for every input sample.

Look at Group Delay Issue

$I_{kk} := \mathrm{Re}\left(H2\left(z_{kk}, b_0, b_1, b_2, b_3\right)\right)$ $\qquad I_0 := 0.01$

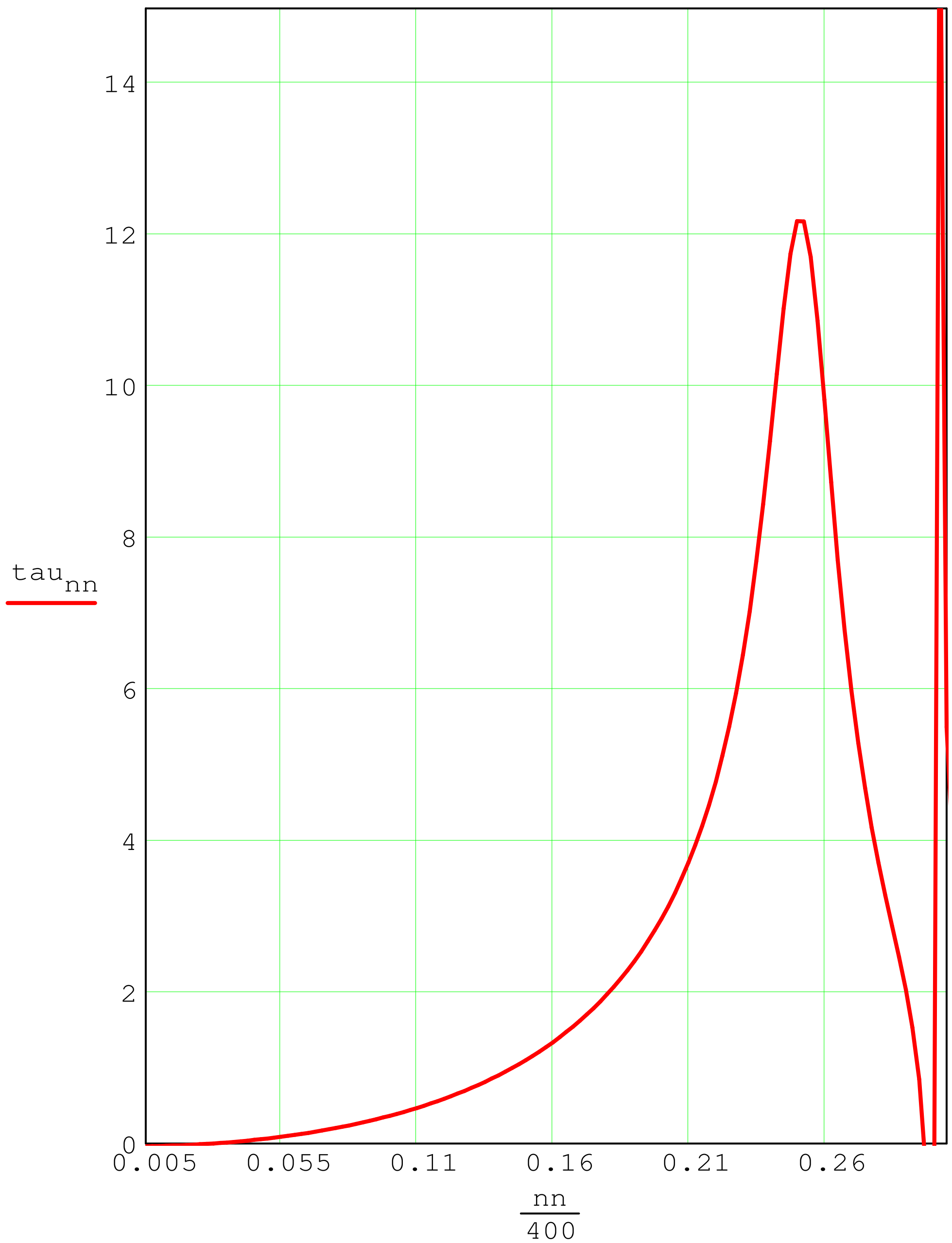
$Q_{kk} := \mathrm{Im}\left(H2\left(z_{kk}, b_0, b_1, b_2, b_3\right)\right)$

$nn := 0 .. \; 198$

$$tau_{nn} := \frac{I_{nn} \cdot Q_{nn+1} - Q_{nn} \cdot I_{nn+1}}{\left[\left(I_{nn}\right)^2 + \left(Q_{nn}\right)^2\right] \cdot delw} \cdot (-1)$$

$tau_5 = 2.409238$

$tau_{nn} := tau_{nn} - 2.445$



In DSP implementation, the number of multiplies is not that much more important than the number of additions. If also, block oriented processing is done, advantages compared to distributed hardware computing can be quite different.

Want to look at using a Kaiser window-based rate 1/2 decimation approach here using FIR filter. This will retain linear phase.

If we exploit the fact that we are going to rate decimate by 1/2 after the filter, can turn this into one-half the computations in the filter. Will think therefore in terms of a 16 tap filter where effectively each input sample results in only 8 taps being computed.

Reference: Theory and Application of Digital Signal Processing, Rabiner, Gold, pp. 94-101

$N := 25$

$pp := 0 .. N-1$

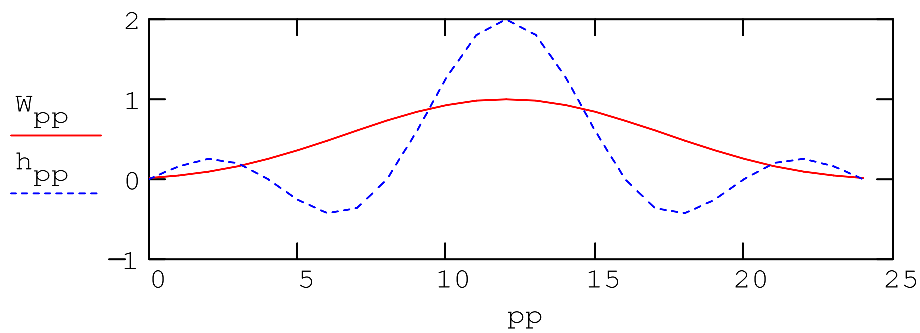
$\beta := 6.0$ $\qquad$ $I0const := I0(\beta)$

$\omega := 2\cdot\pi\cdot 0.125$

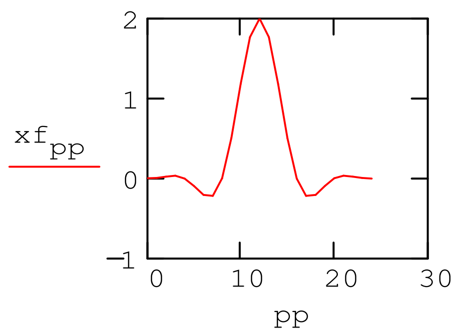
$$ni_{pp} := -\frac{N-1}{2} + pp + 0.001$$

$$W_{pp} := \frac{I0\left[\beta\cdot\sqrt{1-\left[\frac{2\cdot ni_{pp}}{(N-1)}\right]^2}\right]}{I0const} \qquad h_{pp} := \frac{\sin(\omega\cdot ni_{pp})}{0.5\cdot\omega\cdot ni_{pp}}$$

$$h_{\left|\frac{N-1}{2}\right|} := 2$$



$xf_{pp} := W_{pp}\cdot h_{pp}$

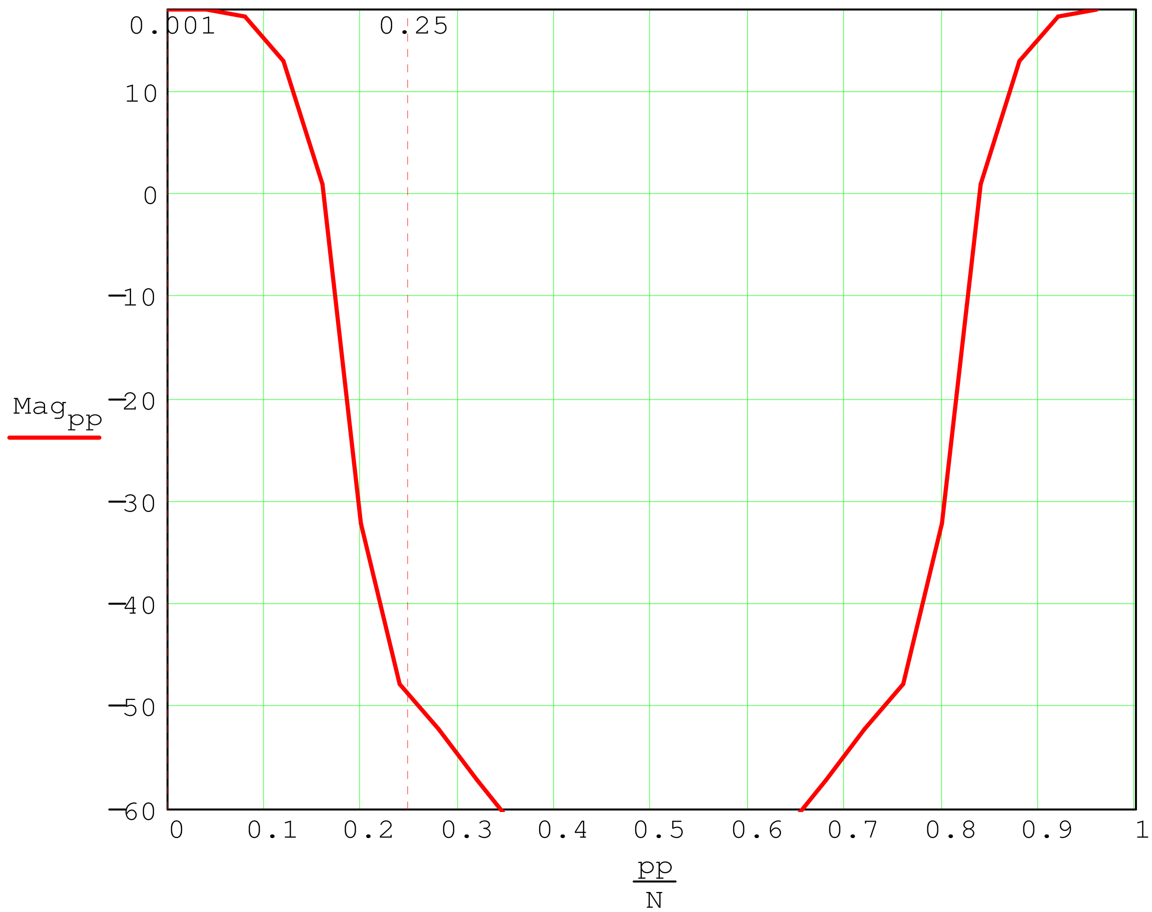


$mm := 0 .. N-1$

$jx = 6.283185i$

$$FT_{pp} := \sum_{mm} \left[xf_{mm} \cdot e^{\left(\frac{jx \cdot mm \cdot pp}{N}\right) \cdot (-1)} \right]$$

$$Mag_{pp} := 10 \cdot \log\left[\left(\left| FT_{pp} \right| \right)^2 \right]$$



xf =

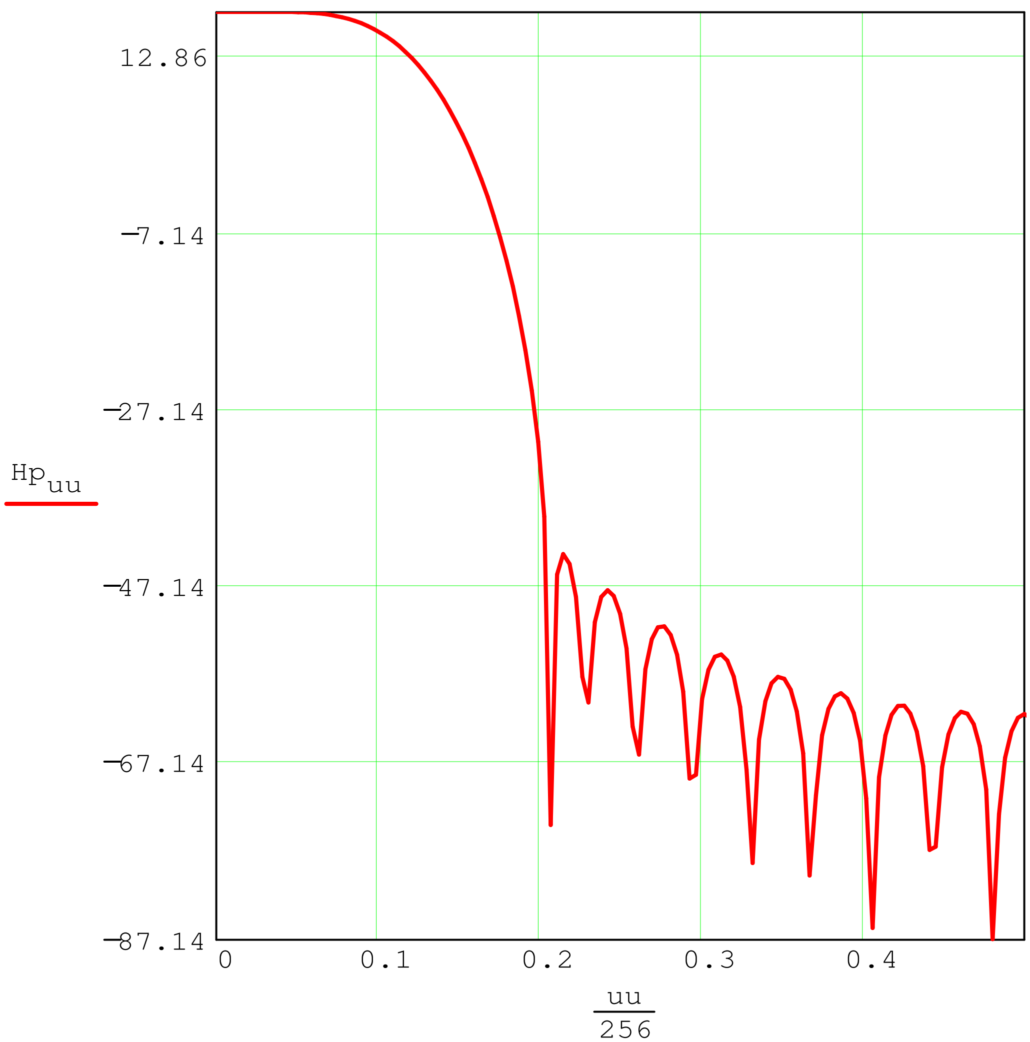
	0
0	2.482816·10-6
1	7.425201·10-3
2	0.02399
3	0.032727
4	-6.34594·10-5
5	-0.093206
6	-0.20506
7	-0.219364
8	3.660978·10-4
9	0.505212
10	1.180138
11	1.767124
12	2
13	1.766231
14	1.178777
15	0.503964

$uu := 0 .. 255$

$zf_{uu} := e^{jx \cdot \frac{uu}{256}}$

$Hb_{uu} := \sum_{pp} \left[xf_{pp} \cdot (zf_{uu})^{-pp} \right]$

$Hp_{uu} := 10 \cdot \log\left[(|Hb_{uu}|)^2 \right]$



After initial rate decimation by 2, can go to larger FIRs

$N := 33$

$pp := 0 .. N-1$

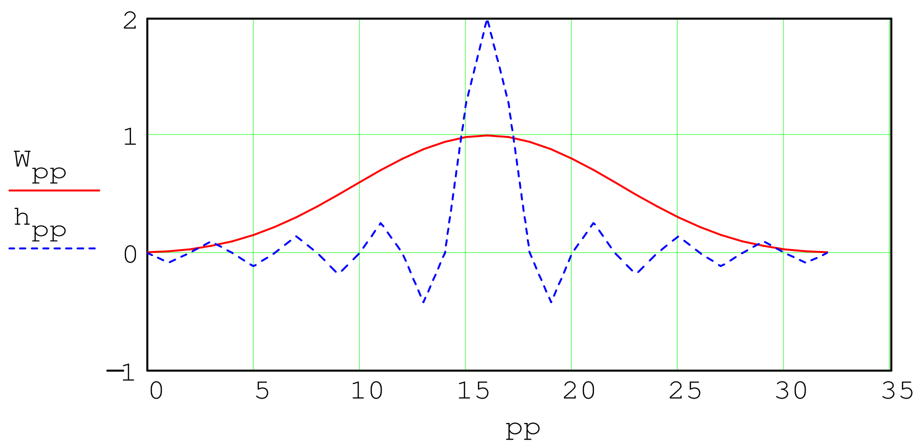
$\beta := 7.5$

$\omega := 2 \cdot \pi \cdot 0.25 \qquad I0const := I0(\beta)$

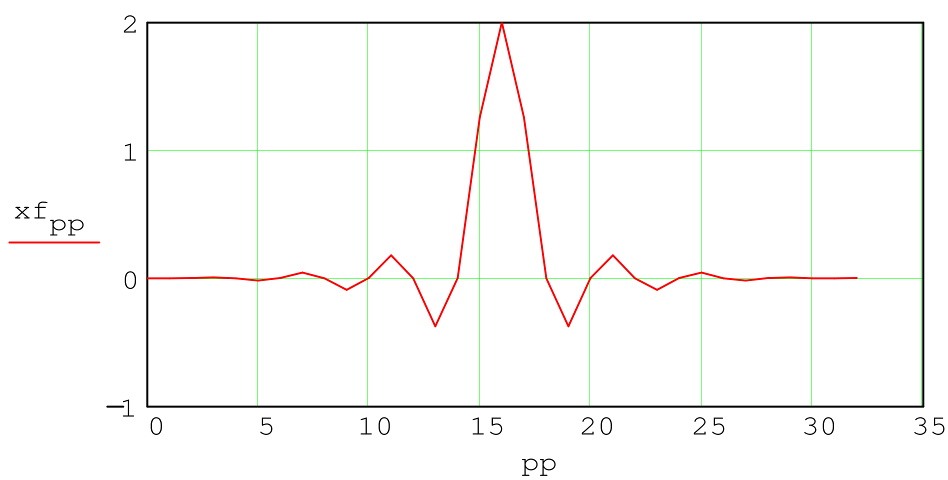
$$ni_{pp} := -\frac{N-1}{2} + pp + 0.0001$$

$$W_{pp} := \frac{I0\left[\beta \cdot \sqrt{1 - \left[\frac{2 \cdot ni_{pp}}{(N-1)}\right]^2}\right]}{I0const} \qquad h_{pp} := \frac{\sin(\omega \cdot ni_{pp})}{0.5 \cdot \omega \cdot ni_{pp}}$$

$$h_{\left|\frac{N-1}{2}\right|} := 2$$



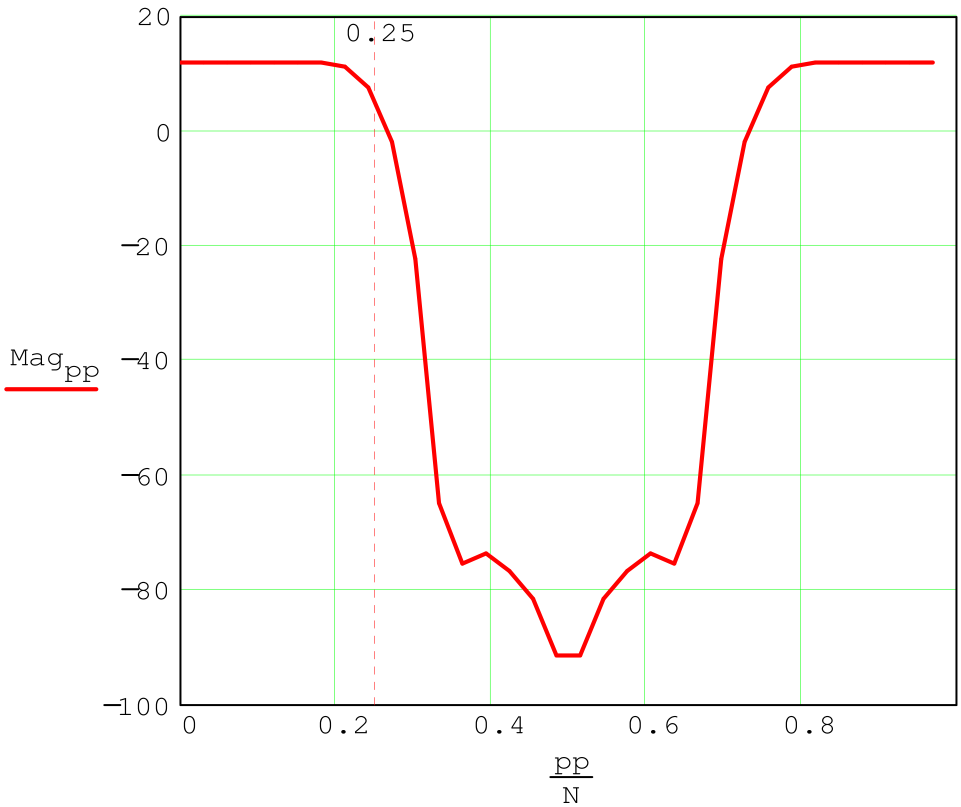
$xf_{pp} := W_{pp} \cdot h_{pp}$



$mm := 0 .. N-1$

$$FT_{pp} := \sum_{mm} \left[xf_{mm} \cdot e^{\left(\frac{jx \cdot mm \cdot pp}{N}\right) \cdot (-1)} \right]$$

$$Mag_{pp} := 10 \cdot \log\left[\left(\left|FT_{pp}\right|\right)^2\right]$$



$N := 13$

$pp := 0 .. N - 1$

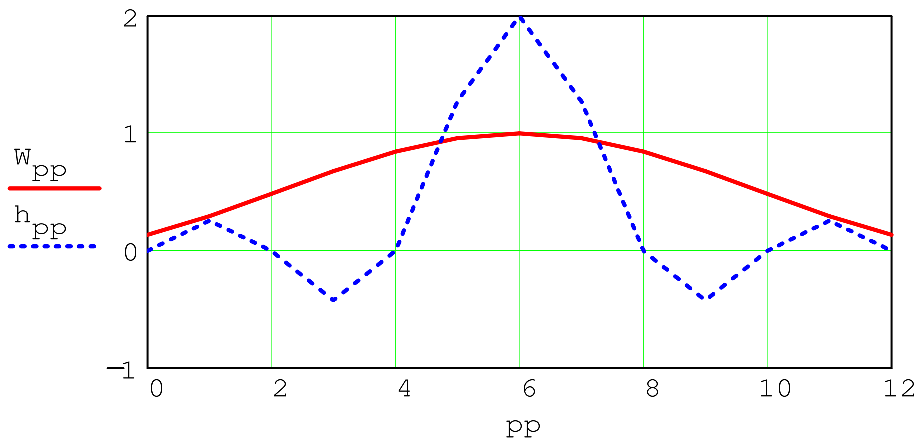
$\beta := 3.5$

$\omega := 2 \cdot \pi \cdot 0.25 \qquad I0const := I0(\beta)$

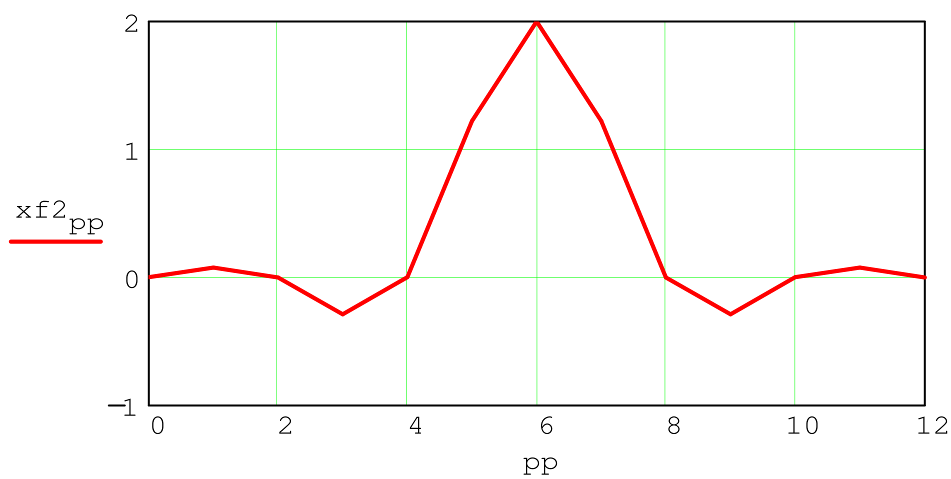
$$ni_{pp} := -\frac{N-1}{2} + pp + 0.0001$$

$$W_{pp} := \frac{I0\left[\beta \cdot \sqrt{1 - \left[\frac{2 \cdot ni_{pp}}{(N-1)}\right]^2}\right]}{I0const} \qquad h_{pp} := \frac{\sin(\omega \cdot ni_{pp})}{0.5 \cdot \omega \cdot ni_{pp}}$$

$$h_{\left|\frac{N-1}{2}\right|} := 2$$



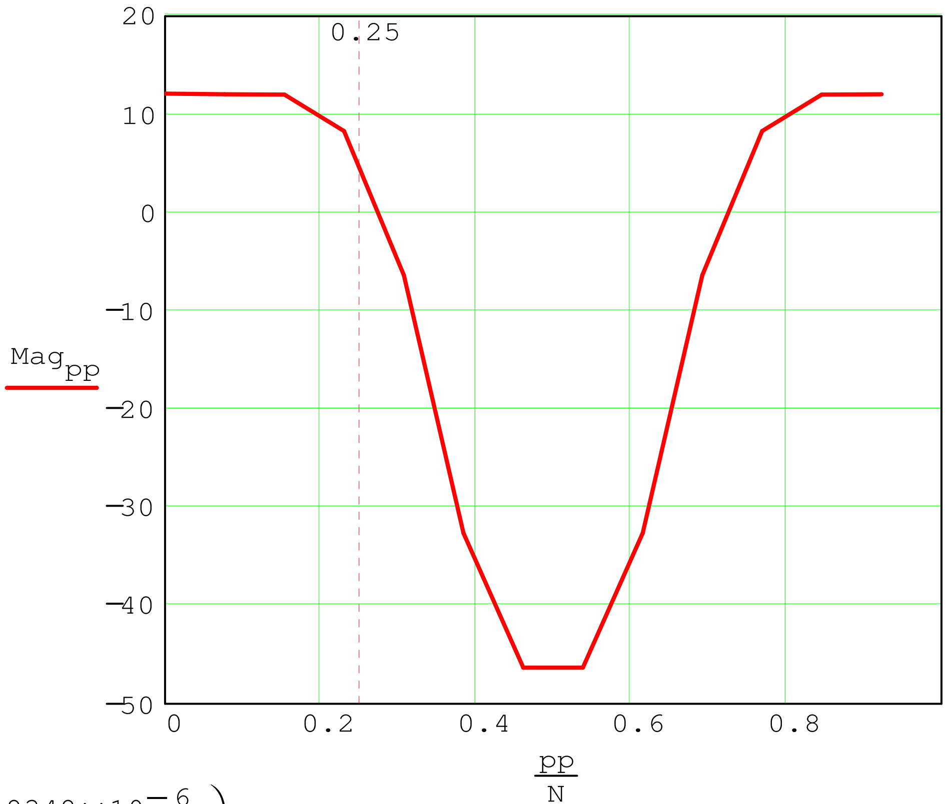
$xf2_{pp} := W_{pp} \cdot h_{pp}$



$mm := 0 .. N - 1$

$$FT_{pp} := \sum_{mm} \left[xf2_{mm} \cdot e^{\left(\frac{jx \cdot mm \cdot pp}{N}\right) \cdot (-1)} \right]$$

$$Mag_{pp} := 10 \cdot \log\left[\left(\left|FT_{pp}\right|\right)^2\right]$$



$$xf2 = \begin{pmatrix} 4.518348 \times 10^{-6} \\ 0.075205 \\ -2.424504 \times 10^{-5} \\ -0.287943 \\ 8.460294 \times 10^{-5} \\ 1.222088 \\ 2 \\ 1.221824 \\ -8.459159 \times 10^{-5} \\ -0.287908 \\ 2.424187 \times 10^{-5} \\ 0.075193 \\ -4.517275 \times 10^{-6} \end{pmatrix} \quad \sum_{pp} \left|xf2_{pp}\right| = 5.170387$$