Examination of Mitchell's I/Q Sampling Scheme

J.A. Crawford

26 April 1993 (Augmented further 29 April)

Context of SW Project (Internal IR&D)

$n := 21$

$Fif := 455 \cdot 10^{3}$

$Fs := \frac{4 \cdot Fif}{(2 \cdot n - 1)}$ Sampling Rate = $Fs = 4.439 \times 10^{4}$

Third-Order Mitchell Algorithm for Hilbert Filter

$a_1 := 0.60323$

$a_2 := 0.12774$

$a_3 := 0.02610$

Reconstructed Q samples using Mitchell's algorithm:

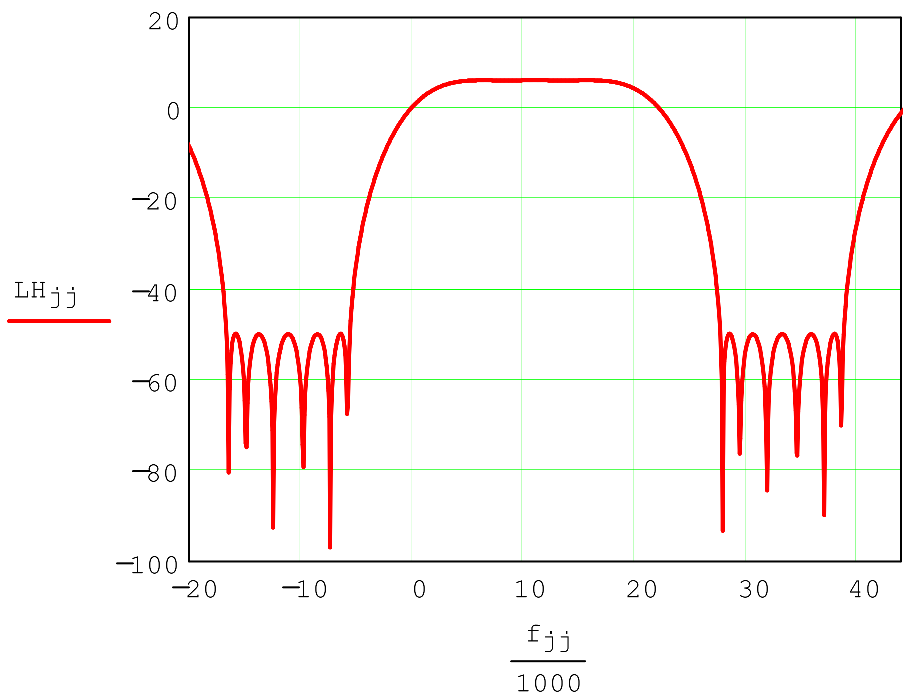
$jj := 1 .. 1500$ $uu := 1 .. 3$

$f_{jj} := jj \cdot 80 - 43000$

$$H_{jj} := 1 + 2 \cdot \sum_{uu} \left[a_{uu} \cdot \sin\left[2 \cdot \pi \cdot (2 \cdot uu - 1) \cdot \frac{f_{jj}}{Fs} \right] \right]$$

$$LH_{jj} := 10 \cdot \log\left[\left(\left| H_{jj} \right| \right)^{2} \right]$$

Filter Transfer Function for Hilbert Transform



Signal being sampled has its spectrum centered at 455 kHz. This center frequency corresponds to 10.25 times the sampling rate. Hence, the spectrum at 455 kHz is aliased down to 0.25 x 44.39 = 11.0975 kHz. Since the waveform being sampled is a real waveform it has a negative spectrum centered at -455 kHz. Upon sampling, this spectrum is aliased up to -11.0976 kHz. In order to perform the Hilbert transform therefore, the filtering must (i) remove the negative frequency components centered at -11 kHz and (ii) downconvert the remaining spectrum to baseband (D.C.). This final translation is accomplished by alternately multiplying the filter outputs with +1 and -1.

As seen above, the negative frequency stopband spans the range of -6 kHz to -16.5 kHz or a total stopband width of about 10 kHz. In this case then, signal bandwidths up to 10 kHz can be accomodated. This 10 kHz of course includes the transition bands of the IF filtering which precedes the bandpass sampling operation. Therefore, realistically, with fairly good filtering ahead of the bandpass sampler, an RF bandwidth of 6 kHz should be achievable.

Fourth-Order Mitchell Algorithm for Hilbert Filter

$a_1 := 0.610741$

$a_2 := 0.144644$

$a_3 := 0.041140$ $a_4 := 0.007475$

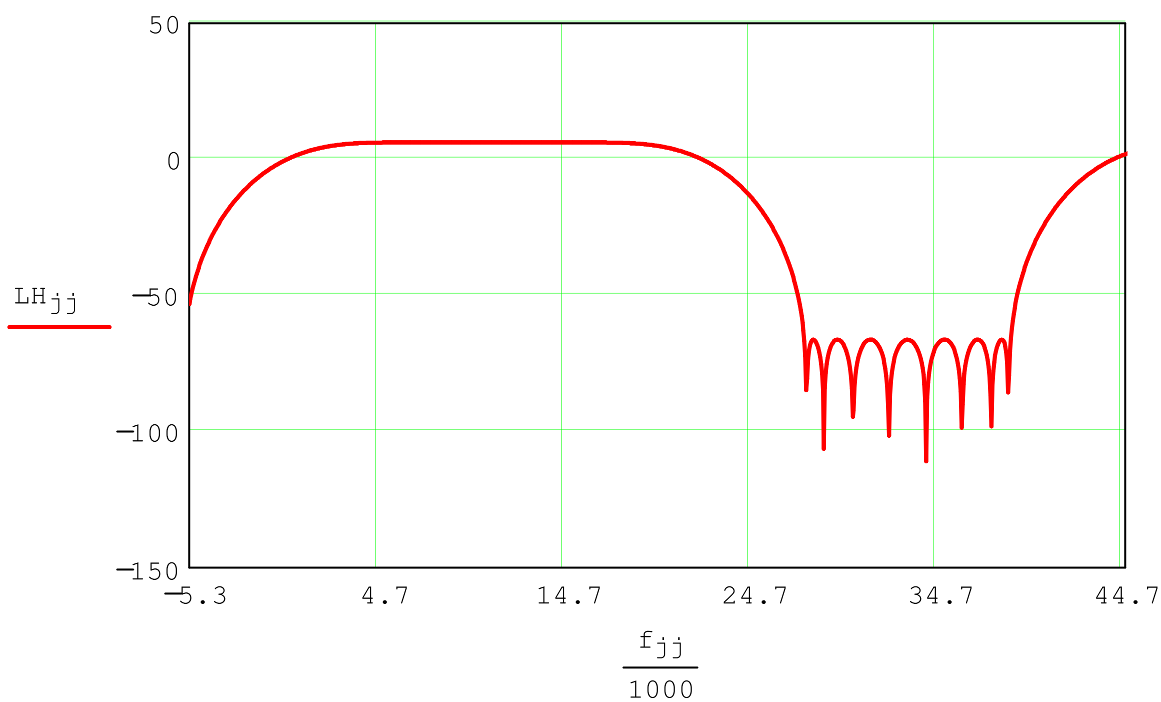
If greater stopband depth is desired, a higher-order algorithm can be employed as shown here.

$jj := 1 .. 1500$ $uu := 1 .. 4$

$f_{jj} := jj \cdot 50 - 5350$

$$H_{jj} := 1 + 2 \cdot \sum_{uu} \left[a_{uu} \cdot \sin\left[2 \cdot \pi \cdot (2 \cdot uu - 1) \cdot \frac{f_{jj}}{Fs} \right] \right]$$

$$LH_{jj} := 10 \cdot \log\left[\left(\left| H_{jj} \right| \right)^2 \right]$$



This approach (4th order) gives back 50 dB image attenuation at its edge for 11 kHz RF bandwidth. Probably run into precision problems more than anything else. In the end, probably best to simply increase the sampling rate somewhat if a higher RF bandwidth is required.

DTR Project Context:

$n := 8$

$Fif := 488 \cdot 10^3$

$Fs := \frac{4 \cdot Fif}{(2 \cdot n - 1)}$ Sampling Rate $= Fs = 1.301 \times 10^5$

Third-Order Mitchell Algorithm for Hilbert Filter

$a_1 := 0.60323$

$a_2 := 0.12774$

$a_3 := 0.02610$

Reconstructed Q samples using Mitchell's algorithm:

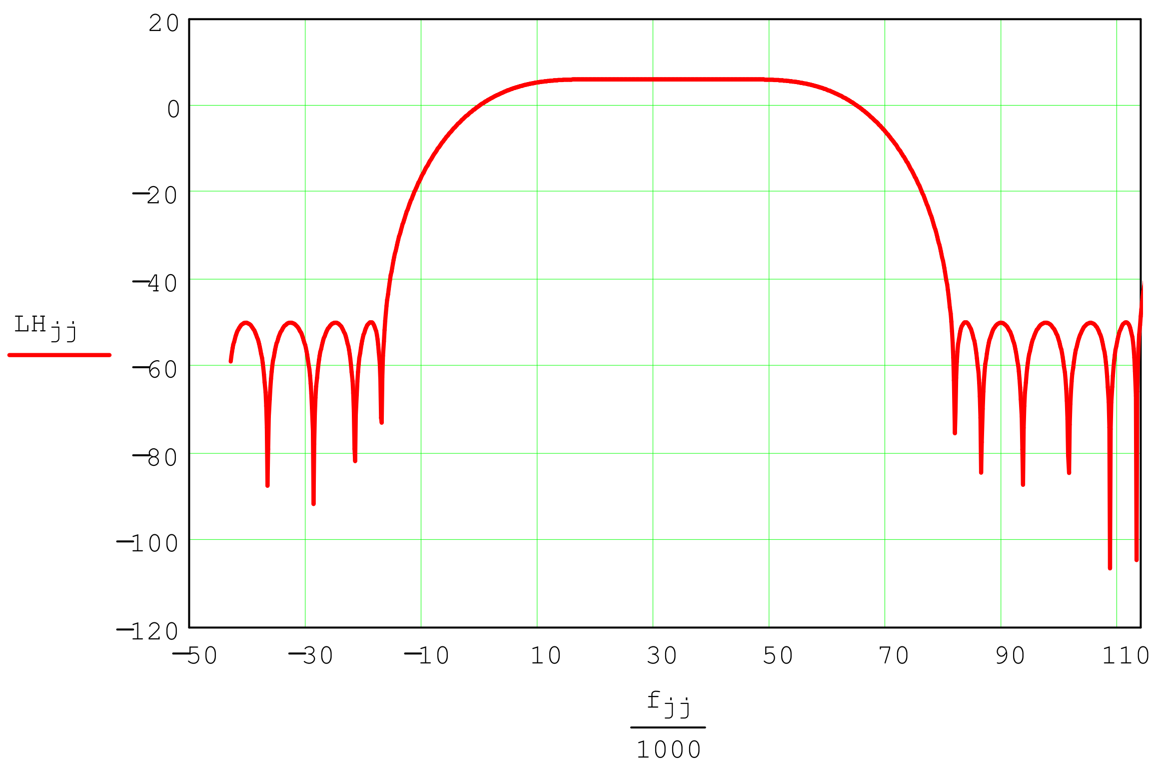
$jj := 1 .. 1500$ $uu := 1 .. 3$

$f_{jj} := jj \cdot 120 - 43000$

$$H_{jj} := 1 + 2 \cdot \sum_{uu} \left[a_{uu} \cdot \sin\left[2 \cdot \pi \cdot (2 \cdot uu - 1) \cdot \frac{f_{jj}}{Fs} \right] \right]$$

$$LH_{jj} := 10 \cdot \log\left[\left(\left| H_{jj} \right| \right)^2 \right]$$

Filter Transfer Function for Hilbert Transform



For the DTR application with this sampling rate, the stopband width is approximately 82 kHz to 114 kHz for positive frequencies corresponding to an available bandlimited RF bandwidth of 32 kHz. Any signal energy falling outside this RF bandwidth will result in degraded stopband performance and hence erroneous I/Q information extraction.

Trying a little higher sampling rate:

$n := 7 \quad Fif := 488 \cdot 10^3$

$Fs := \frac{4 \cdot Fif}{(2 \cdot n - 1)}$ Sampling Rate = $Fs = 1.502 \times 10^5$

Reconstructed Q samples using Mitchell's algorithm:

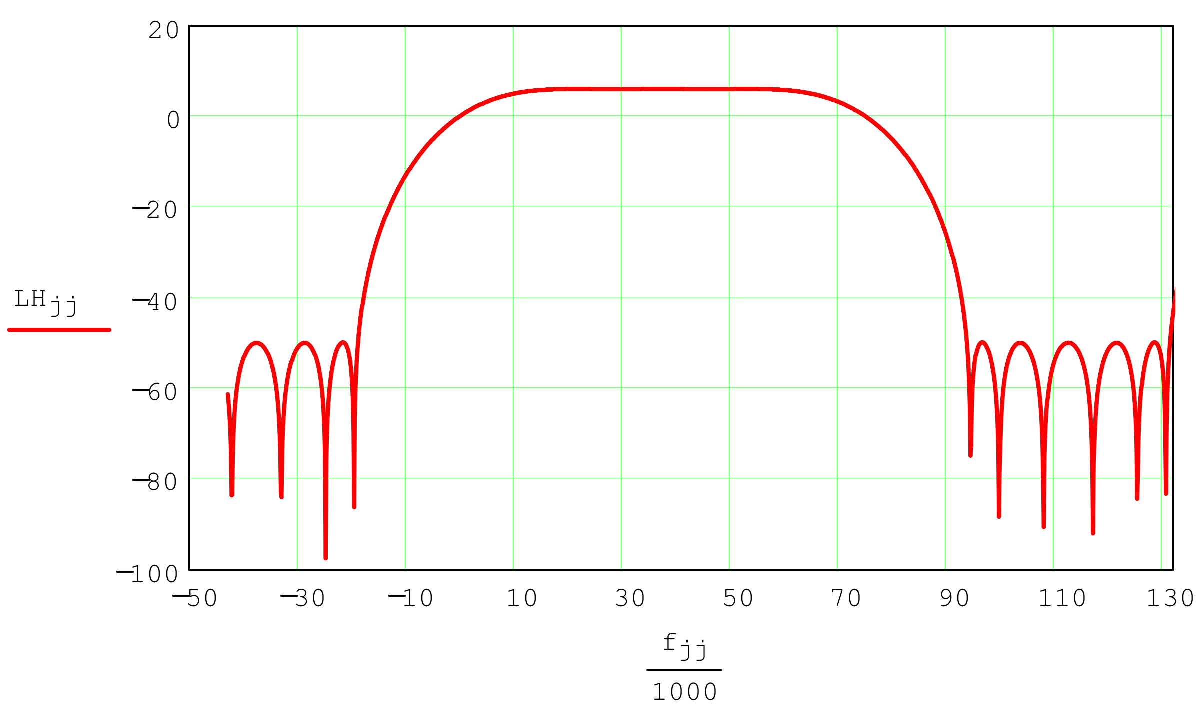
$jj := 1 .. 1500$

$f_{jj} := jj \cdot 120 - 43000$

$$H_{jj} := 1 + 2 \cdot \sum_{uu} \left[a_{uu} \cdot \sin\left[2 \cdot \pi \cdot (2 \cdot uu - 1) \cdot \frac{f_{jj}}{Fs} \right] \right]$$

$$LH_{jj} := 10 \cdot \log\left[\left(\left| H_{jj} \right| \right)^2 \right]$$

Filter Transfer Function for Hilbert Transform



For this higher sampling rate, the permissible RF bandwidth is roughly (95-132 kHz) 37 kHz. This would be adequate for any reasonable 25 kHz channel spacing situation. So if necessary, this could be adopted in the DSP resource.

Softwave Case

$n := 13 \quad Fif := 271.1111 \cdot 10^{3}$

$Fs := \dfrac{4 \cdot Fif}{(2 \cdot n - 1)}$ Sampling Rate $Fs = 4.338 \times 10^{4}$

Reconstructed Q samples using Mitchell's algorithm:

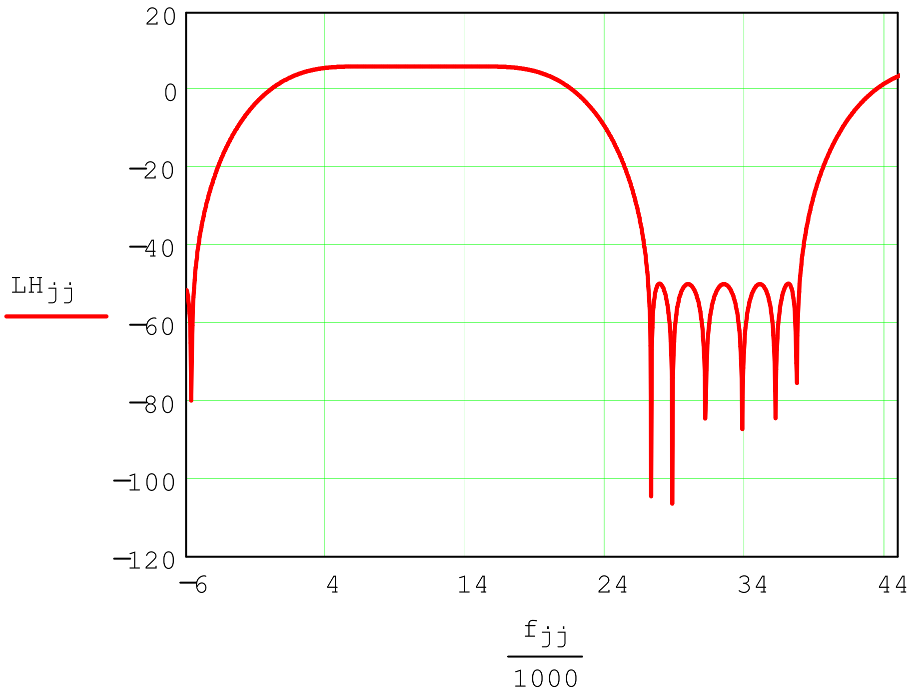
$jj := 1 .. 1500$

$f_{jj} := jj \cdot 40 - 6000$

$$H_{jj} := 1 + 2 \cdot \sum_{uu} \left[a_{uu} \cdot \sin\left[2 \cdot \pi \cdot (2 \cdot uu - 1) \cdot \frac{f_{jj}}{Fs} \right] \right]$$

$$LH_{jj} := 10 \cdot \log\left[\left(\left| H_{jj} \right| \right)^{2} \right]$$

Filter Transfer Function for Hilbert Transform



In this case, the stopband width remains a bit larger than 10 kHz, so for 6 kHz channel operation, this is quite adequate.