PHASE NOISE MEASUREMENT USING A HIGH RESOLUTION COUNTER WITH ON-LINE DATA PROCESSING

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Summary

The measurement of close-in phase noise using time domain techniques will be discussed. A review of the theoretical basis for the measurements and some extensions will be presented. A system for making the measurements using a high resolution reciprocal counter and a desk top calculator will be described. The capabilities of the system will be reviewed.

Introduction

The measurement of phase noise can be made by a variety of methods which are described in the current literature. However, if one desires to measure the spectral characteristics close into the carrier, the task becomes difficult and time consuming. Measurements down to the 10 to 100 Hz range can be accomplished with spectrum or wave analyzers depending on the noise level. Measurements much below these offset frequencies are virtually impossible with these techniques.

The techniques to make measurements closer to the carrier usually resort to time domain analysis which is also described in the current literature. The primary impediment to the utilization of these techniques has been the lack of equipment to make it easy to use. Recent equipment advances such as programmable instruments and versatile calculators, for example, have provided the means to implement these techniques.

Before describing the measurement system, a review of the theoretical basis for the measurement will be presented. Most of this review will follow the standard literature with some exceptions that will tend to put more emphasis on the calculator utilization.

A general transfer function will be derived for the counter-calculator system which should cover a standard frequency measurement or comparison of the phase between two oscillators or the phase of an oscillator compared to itself through a delay line.

For a constant gate time (or constant delay), the transfer function can be written as the product of two functions, one controlled by the counter (or delay line) and the other by the calculator.

We prove that a linear combination of frequency measurements is equivalent to a filter which can be adjusted by proper choice of the linear combination coefficients. A particular choice of coefficients is assumed for which we derive closed formulas for the continuous and the delta function (bright line) spectral densities. The minimum spectral level due to counter quantization is derived.

Derivation of Time Domain Relationships to Phase Spectra Characteristics

Definitions and Notation

Let us begin with a brief review of random variables, linear systems, notations and definitions.

Let x(t) be a real random variable. The auto correlation function of x(t) is defined as:

$$R_x(t,\tau) = \langle x(t) \cdot x(t+\tau) \rangle \qquad (1)$$

where $\langle\ \rangle$ means statistical or ensemble average.

If x(t) is stationary in the wide sense, then the auto correlation does not depend on t and we can write:

$$R_x(\tau) = \langle x(t) \cdot x(t+\tau) \rangle \qquad (2)$$

If x(t) is ergodic, then the statistical average can be replaced by time average.

We will assume without any loss of generality that the mean value of x(t) is zero. In this case we have:

$$R_x(0) = \langle x^2(t) \rangle = \sigma_x^2 \qquad (3)$$

The spectral density of x(t) is defined as the Fourier transform of its auto correlation function:

$$S_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j\omega\tau} dt \qquad (4)$$

The inversion formula gives:

$$R_x(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega)\, e^{+j\omega\tau} d\omega \qquad (5)$$

Let a linear time invariant system with transfer function H(ω) be driven by the random variable x(t). The output of the system represented by m(t) will be also a random variable as in Fig. 1.

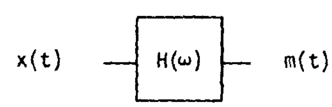

Figure 1

It can be shown that:

$$S_m(\omega) = \left| H(\omega) \right|^2 S_x(\omega) \qquad (6)$$

Equation 6 will be used to determine $S_x(\omega)$ in the next section. For more details see references 1,2 and 3.

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Measuring Spectral Density

Normally a narrow band filter represented by $H(\omega)$ is used to determine the spectra of the random variable $x(t)$ as explained below. Two special cases are considered.

A. Continuous Spectra - The spectral density can be considered approximately constant over the filter bandwidth and we can write:

$$\sigma_m^2 = R_m(0) \cong S_x(\omega_0) \frac{1}{\pi} \int_{\omega_1}^{\omega_2} |H(\omega)|^2 d\omega$$

$$\cong \frac{1}{\pi} |H(\omega_0)|^2 \cdot (\omega_2 - \omega_1) \cdot S_x(\omega_0) \tag{7}$$

where ω_0 is the filter center frequency and ω_1, ω_2 define the filter band. (We have used the fact that $|H(\omega)|$ and $S_x(\omega)$ are even functions of ω.)

Solving for the spectral density we get:

$$S_x(\omega_0) \cong \frac{\sigma_m^2}{\frac{1}{\pi}\int_{\omega_1}^{\omega_2} |H(\omega)|^2 d\omega} \cong \frac{\sigma_m^2}{\frac{1}{\pi}|H(\omega_0)|^2 (\omega_2 - \omega_1)} \tag{8}$$

Where σ_m^2 can be determined with an average power meter or using samples of K measurements of m as:[1,4,5]

$$\sigma_m^2 = \left\langle \frac{1}{K-1} \sum_{i=1}^{K} (m_i - \frac{1}{K} \sum_{j=1}^{K} m_j)^2 \right\rangle$$

$$= \left\langle \frac{1}{K-1} \left[(\sum_{i=1}^{K} m_i^2) - \frac{1}{K} (\sum_{j=1}^{K} m_j)^2 \right] \right\rangle^* \tag{9}$$

If we use only one sample of K measurements of $m(t)$, we get an estimation of σ_m^2 which is satisfactory for most cases.

B. The Spectra Contains Delta Functions. This is the case when $x(t)$ contains periodic terms, then the result of the integration is no longer dependent on the filter bandwidth.

For example let $x(t)$ be given by:

$$x(t) = x_p \cdot \cos \omega_0 t = \frac{1}{2} x_p [e^{j\omega_0 t} + e^{-j\omega_0 t}] \tag{10}$$

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*For complex $m(t)$, σ_m^2 is given by:

$$\sigma_m^2 = \left\langle \frac{1}{K-1} \sum_{i=1}^{K} |m_i - \frac{1}{K} \sum_{j=1}^{K} m_j|^2 \right\rangle$$

$$= \left\langle \frac{1}{K-1} \left[(\sum_{i=1}^{K} |m_i|^2) - \frac{1}{K} |\sum_{j=1}^{K} m_j|^2 \right] \right\rangle$$

We have:

$$S_x(\omega) = 2\pi (\frac{x_p}{2})^2 [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \tag{11}$$

Which gives:

$$\sigma_m^2 = |H(\omega_0)|^2 \frac{1}{2} x_p^2 \tag{12}$$

Note that $\frac{1}{2} x_p^2$ is the average power of $x(t)$.

It should be pointed out that during all the derivations we will use the "two-sided" spectral density, that is the integrations will be from $-\infty$ to $+\infty$. This will keep our Fourier transform pairs in the standard notation used by most EE's. Fourier frequency will be in radians per seconds.

Frequency Counter as a Linear System

Consider the signal

$$V(t) = V_0 \cdot \cos(2\pi\gamma_0 t + \theta_0 + \theta(t)) \tag{13}$$

where V_0 and γ_0 are the nominal amplitude and frequency and $\theta(t)$ represents a random phase variation. θ_0 is chosen such that $\langle\theta(t)\rangle = 0$.

Any amplitude variation is assumed to be eliminated by limiters or some other method and for this reason is disregarded.

The signal phase is defined as:

$$\phi(t) = 2\pi\gamma_0 t + \theta_0 + \theta(t) \tag{14}$$

The signal frequency $\nu(t)$ and the angular frequency $\Omega(t)$ are related by:

$$2\pi\nu(t) = \Omega(t) = \frac{d\phi}{dt} \tag{15}$$

and are assumed to be positive for all t, which is equivalent to $\phi(t)$ be a monotonic increasing function of time, that is, $2\pi\nu_0 - \theta(t) > 0$.

Counter Model

An ideal counter can be modeled as a system that measures phase variation over an interval of time τ, called gate time, and divides the result by $2\pi\tau$.

Let $\nu(t)$ represent the result of a counter measurement, then have:

$$\nu(t) = \frac{1}{2\pi\tau} \cdot [\phi(t) - \phi(t-\tau)] \tag{16}$$

Taking the Fourier transform from both sides and using the shifting theorem, we have:

$$\Gamma(\omega) = \frac{1}{2\pi\tau} [\Phi(\omega) - \Phi(\omega) e^{-j\omega\tau}] \tag{17}$$

Here the upper case letters are used to represent the Fourier transform.

Which can be reduced to:

$$\Gamma(\omega) = \frac{\omega}{2\pi} \cdot \frac{\sin \omega \tau/2}{\omega \tau/2} \cdot e^{-j\omega\tau/2} \cdot \Phi(\omega) \qquad (18)$$

We may conclude that the counter is a linear system with transfer function (or gain) given by:

$$H_\phi(\omega) = \frac{\omega}{2\pi} \frac{\sin \omega \tau/2}{\omega \tau/2} \cdot e^{-j\omega\tau/2} \qquad (19)$$

Where the index ϕ in transfer function means that it applies for the phase as input.

The transfer function when the angular frequency $\Omega(t)$ is considered as input is given by:

$$H_\Omega(\omega) = \frac{1}{j\omega} H_\phi(\omega) = \frac{1}{2\pi j} \cdot \frac{\sin \omega\tau/2}{\omega \tau/2} e^{-j\omega\tau/2} \qquad (20)$$

Practical Counters

Below are some problems that we might incur due to practical considerations.

In reality a counter detects and counts zero crossings of the signal being measured. If the phase of the input signal is a monotonic increasing function of time, as it was assumed, then the model is quite satisfactory.

On the other hand if the phase of the input signal can decrease, which is equivalent to a negative frequency, then we may get extra counts, for example in the vector diagram representation of the signal, Fig. 2. Every time that the resultant vector crosses the Y axis from right to left, we get a count; if the vector goes back we will get an extra count.

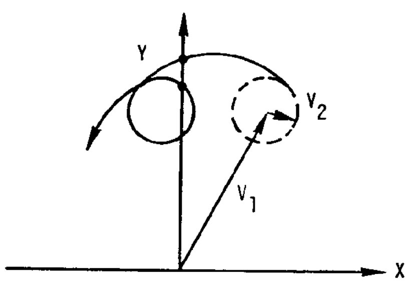

Figure 2

In Fig. 2, V_1 represents a sine wave and V_2 a interference that causes phase modulation.

Practical counters also have quantization problems, that is the number of counts is an integer and any fraction of 2π in the phase variation is disregarded. This problem is reduced by reciprocal counters with a high frequency clock. In a reciprocal counter, the actual gate time is a multiple of the signal period and if the phase fluctuation is not too large ($\dot{\theta}(t) \ll 2\pi\nu_0$), then the gate time can be assumed constant as far as the counter transfer function is concerned.

The last problem in a real counter is the sampling, that is the counter output is not a continuous variable but a sequence of numbers obtained at the end of each gate time. A special case solution is presented in reference 5, Appendix 1.

Modifying the Counter Transfer Function

In order to measure spectral density, we want the counter to look like a very narrow band filter, ideally we want a delta function.

A weighted combination of a sequence of measurements taken at different times will give:

$$m(t) = \sum_{i=0}^{M} \alpha_i \cdot \nu_i \qquad (21)$$

where $\nu_i = \nu(t_i)$

Let Fig. 3 represent the phase as a function of time, τ_i the gate time and t_i the time at the end of the i reading. The last reading is taken at the time which is considered to be the current time, that is $t_0 = t$.

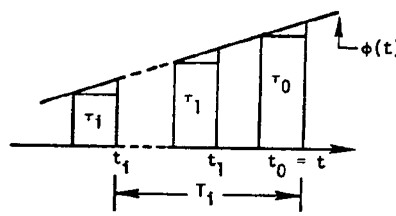

Figure 3

Replacing Y(t) as function of $\phi(t)$ and defining $T_i = t_0 - t_i$ we have:

$$m(t) = \frac{\alpha_0}{2\pi\tau_0} \left[\phi(t) - \phi(t-\tau_0)\right] + \ldots + \frac{\alpha_M}{2\pi\tau_M} \left[\phi(t-T_M) - \phi(t-T_M-\tau_M)\right] \qquad (22)$$

Taking the Fourier transform we get the transfer function:

$$H_\phi(\omega) = \left[\frac{\alpha_0}{2\pi\tau_0} (1-e^{-j\omega\tau_0}) + \ldots + \frac{\alpha_M}{2\pi\tau_M} e^{-j\omega T_M} (1-e^{-j\omega\tau_M})\right] \qquad (23)$$

The counter calculator equivalent transfer function can be adjusted to approximate the desired filter transfer function by proper choice of α_i, τ_i and T_i. The weights α_i can be complex and their phase is equivalent to a change in the time the measurements are made, which can be used to simulate a variable time between measurements.

If we make the gate time τ_i a constant τ and the time between consecutive measurements, defined as dead time, a constant τ_d we have:

$$H_\phi(\omega) = \left[\alpha_0+\alpha_1 e^{-j\omega(\tau+\tau_d)}+\ldots+\alpha_M e^{-jM\omega(\tau+\tau_d)}\right] \cdot \left[\frac{\omega}{2\pi}\cdot\frac{\sin\omega\tau/2}{\omega\tau/2}\cdot e^{-j\omega\tau/2}\right] \qquad (24)$$

The first bracket, which is mainly determined by the calculator via the α_S, can be interpreted as a truncated complex Fourier series and its terms can be adjusted as a series representation of the desired filter function[6,7]. The second bracket can be considered as a fixed filter and for large M the first bracket completely determines H_ϕ.

Other types of measurement such as comparing the phase of two oscillators of same frequency or comparing the phase of an oscillator to itself, using a delay line, will have the same type of transfer function due to the fact that time differences are equivalent to phase differences.

In the case of a fixed delay line, we use the calculator controlled part of the transfer function to achieve the desired filter characteristic.

A particular but very useful choice[7] is $\alpha_i = (-1)^i$ which reduces m(t) to:

$$m(t) = (y_0 - y_1) + (y_2 - y_3) + \ldots + (y_{M-1} - y_M) \qquad (25)$$

Let $N = (M+1)/2$ be the number of pairs of measurements represented by the terms in parenthesis, then we can reduce H_ϕ to:

$$H_\phi(\omega) = \left[1+e^{-j2\omega(\tau+\tau_d)} + \ldots e^{-j(N-1)2\omega(\tau+\tau_d)}\right] \cdot 2\sin\left(\omega\frac{\tau+\tau_d}{2}\right)\cdot\frac{\omega}{2\pi}\,\frac{\sin\omega\tau/2}{\omega\tau/2}\, e^{-j\omega(\tau+\tau_{d/2})} \qquad (26)$$

The bracket can be recognized as a geometric series which gives:

$$H_\phi(\omega) = \frac{\omega}{\pi}\cdot\frac{\sin\omega\tau/2}{\omega\tau/2}\cdot\sin\left(\omega\frac{\tau+\tau_d}{2}\right)\cdot\frac{\sin N\omega(\tau+\tau_d)}{\sin\omega(\tau+\tau_d)}\cdot e^{-j\omega[N(\tau+\tau_d)-\tau_{d/2}]} \qquad (27)$$

A very important point to be kept in mind is that in general this corresponds to a filter with many pass bands.

For most practical cases, the phase noise is a fast decreasing function of ω and only the first pass band of H_ϕ has to be considered, as in references 6 and 7, which have plots of $|H_\Omega(\omega)|$ which is equal to $|H_\phi(\omega)/\omega|$. Error will occur depending on how fast the phase noise decreases as a function of ω such as ω^{-3}, ω^{-2}, ω^0 = constant, and we may have to consider more than one pass band.[6,7]

Calculation of Spectra Density

So far we have proven that the linear combination of the frequency measurements m(t) can be considered as the output of a filter whose input is $\phi(t)$ and transfer function H_ϕ or input $\Omega(t)$ and transfer function $H_\Omega(\omega)$. This is represented in the block diagram, Fig. 4.

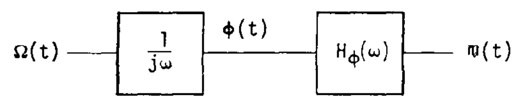

Figure 4

As in the linear system we have two cases.

A. Continuous Spectra - The spectra can be considered as constant over the filter bandwidth. Using the fact that $\alpha_m^2 = R_m(0)$ and eq. 5, we have:

$$\sigma_m^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|H_\phi(\omega)\right|^2 S_\phi(\omega)d\omega \qquad (28)$$

If we use $H_\Omega(\omega)$ instead of $H_\phi(\omega)$, we have:

$$\sigma_m^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|H_\Omega(\omega)\right|^2 \omega^2\, S_\phi(\omega)\, d\omega \qquad (29)$$

A normalized expression in ω for $|H_\phi|$ is given as:

$$|H_\phi(\omega)| = \left|\frac{N\omega}{\pi}\cdot\frac{\sin r\frac{\pi}{2}\frac{\omega}{\omega_0}}{r\frac{\pi}{2}\frac{\omega}{\omega_0}}\cdot\sin\pi/2\,\frac{\omega}{\omega_0}\cdot\frac{\sin N\pi\frac{\omega\pm\omega_0}{\omega_0}}{N\sin\pi\frac{\omega\pm\omega_0}{\omega_0}}\right| \qquad (30)$$

Where r and ω_0 are:

$$r = \frac{\tau}{\tau+\tau_d} \qquad (31)$$

$$\omega_0 = \frac{\pi}{\tau+\tau_d} \qquad (32)$$

We have used the fact that sin(x) only changes the sign when we add or subtract π to x to obtain the term $(\omega \pm \omega_0)/\omega_0$. For large N and $|\omega \pm \omega_0| << \omega_0$ we can approximate $|H_\phi|$ as:

$$|H_\phi| \cong \left| \frac{N\omega_0}{\pi} \cdot \frac{\sin r\pi/2}{r\pi/2} \cdot \frac{\sin N\pi \frac{\omega \pm \omega_0}{\omega_0}}{N\pi \frac{\omega \pm \omega_0}{\omega_0}} \right| \quad (33)$$

Note that $|H_\phi|$ is equal to a constant times $|\sin(x)/x|$ centered at $+\omega_0$ and $-\omega_0$. Solving the integral (28), we have:

$$S_\phi(\omega_0) \cong \frac{\sigma_m^2}{\frac{1}{\pi} \left| H_\phi(\omega_0) \right|^2 \frac{\omega_0}{N}} = \frac{1}{8} \left(\frac{r \frac{\pi}{2}}{\sin r \frac{\pi}{2}} \right)^2 \frac{\sigma_m^2}{N f_0^3} \quad (34)$$

Where ω_0/N can be interpreted as the equivalent bandwidth in radians per second (see Fig. 5) and $f_0 = \omega_0/2\pi$.

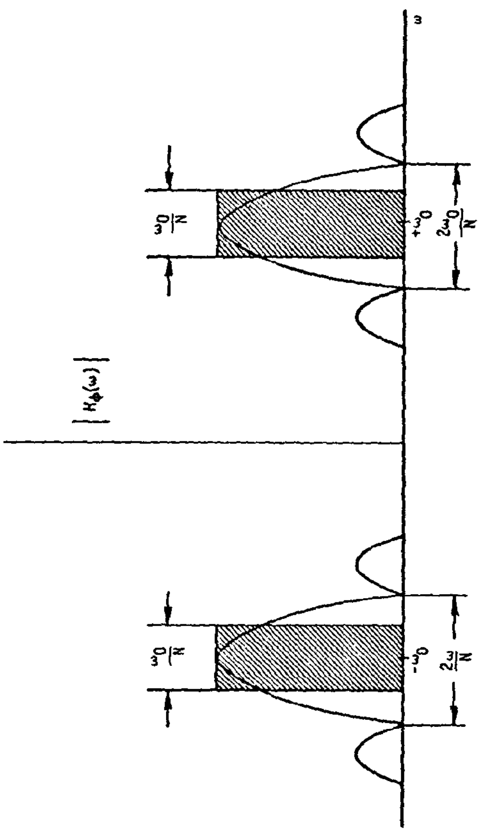

Figure 5

The shaded area in Fig. 5 represents the equivalent ideal filter.

If we select $\tau_d = \frac{1}{2}\tau$, then the multiple responses of $H_\phi(\omega)$ do not begin until $5\omega_0$ is reached. See figure 6.

$$S_\phi(\omega_0) \cong \frac{1}{8} \left(\frac{\pi/3}{\sin \pi/3}\right)^2 \frac{\sigma_m^2}{N f_0^3} \quad (35)$$

$$S_\phi(\omega_0) \cong .183 \frac{\sigma_m^2}{N f_0^3} \quad (36)$$

The phase spectral density due to the pass band centered at ω_0 can be determined by equation (36) where σ_m^2 can be determined by numerical methods.

<u>B. Power Spectra Contains Delta Function (Bright Line)</u>. Let us consider the special case of a phase modulated signal such as

$$V = V_0 \cos(\Omega_0 t + \theta_p \cos \omega_0 t) \quad (37)$$

The phase spectra for the modulation is:

$$S_\theta(\omega) = 2\pi \left(\tfrac{1}{2}\theta_p\right)^2 [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$= \left(\tfrac{1}{2}\theta_p\right)^2 [\delta(f - f_0) + \delta(f + f_0)] \quad (38)$$

The signal can be expanded in the usual form using Bessel functions and for small θ_p we have:

$$J_0(\theta_p) \cong 1$$

$$J_1(\theta_p) \cong 1/2\ \theta_p$$

$$\mathcal{L}(f) \cong \left(\tfrac{1}{2}\theta_p\right)^2 \delta(f - f_0) \quad (39)$$

Where $\mathcal{L}(f)$ is defined as the energy at $\Omega_0 + \omega_0$ per Hz divided by the total energy.

From the previous equations (37, 38), we conclude that $\mathcal{L}(f)$ and the "two-sided" phase noise S_ϕ are approximately equal for small θ_p.

The relation between σ_m^2 and θ_p is obtained by solving the integral (28) giving:

$$\left(\tfrac{1}{2}\theta_p\right)^2 = \frac{\sigma_m^2}{2\left|H_\phi(\omega_0)\right|^2} = \left[\frac{\sigma_m^2}{\frac{1}{\pi}\left|H_\phi(\omega_0)\right|^2 \frac{\omega_0}{N}} \right] \left(\frac{f_0}{N}\right) \quad (40)$$

From the previous equation, we conclude that we can determine the phase spectrum as for the continuous case and multiply the result by f_0/N, to obtain $(\frac{1}{2}\theta_p)^2$.

The presence of delta functions (bright lines) can easily be detected due to the fact that σ_m^2 is not dependent on N, as is indicated by the equation relating σ_m^2 and $(\frac{1}{2}\theta_p)^2$.

So far we have relations to determine the phase spectral density $S_\phi(\omega)$ and the bright lines intensity $(\frac{1}{2}\theta_p)^2$. In the next section, we will derive the system resolution using a statistical approach to the quantization problem.

Estimation of the System Resolution

The system resolution is determined by the minimum time variation that the reciprocal counter can resolve, which is the clock period.

Let τ_c be the clock period, τ the gate time and ν_i the frequency of the input signal.

We will assume that the noise of the input signal will cause a maximum time variation of $\ell \tau_c$ during the gate time. Let $\delta\nu$ represent the resultant frequency variation then we have:

$$\delta\nu = \frac{\tau_c}{\tau} \nu_i \tag{41}$$

If we assume that ℓ is uniformly distributed from -1 to +1 and that the resultant $\delta\nu$ in each frequency measurement is independent of all previous ones, than m(t) is the sum of independent uniformly distributed random variables.

Using the central limit theorem, we concluded that for large N the random variable m(t) is approximately Gaussian with variance equal to the sum of the variances for each frequency measurement resulting in:

$$\sigma_m^2 = 2N\left[\frac{1}{3}\left(\frac{\tau_c}{\tau} \nu_i\right)^2\right] \tag{42}$$

where 2N is the number of frequency measurements and the term in brackets is the variance of $\delta\nu$.

This implies in a resolution given by:

$$S_\phi(\omega_0) \cong \left(\frac{r\pi/2}{\sin r\pi/2}\right)^2 \frac{\tau_c}{3r^2} \frac{\nu_i^2}{f_0} \tag{43}$$

The Measurement System

System Description

The system consists of five major components: a high resolution reciprocal counter (HP 5345A) with external gating capabilities, a measurement storage plug-on unit (HP 5358A), a mixer/amplifier unit (HP 10830A), a desk top calculator (HP 9825A) and printer/plotter output device (HP 9871A) as shown in figure 7. Communication and control between the various instruments is provided by a digital interface system (HP-Interface Bus). In addition, a test tone generator (HP 10831A) and time of day clock are included to enhance the system's capabilities. A functional block diagram is shown in figure 8.

The counter provides the system with the ability to make high resolution (2 nsec) period or time interval measurement or frequency (by the reciprocal technique). This determines the system's sensitivity floor limit as will be shown later. In addition, the counter has the ability to be gated from an external sample time signal. This is necessary in order to utilize various sampling functions and thus determine the offset frequency at which a spectra measurement is made.

The counter by itself does not have sufficient control or data output capacity, so a measurement storage plug in unit is included. It extends several capabilities of the counter. The front panel gate times are (as in most counters) in decade steps. This does not allow enough flexibility for spectra characterization so a gate generator is incorporated in the plug-in which generates a measurement time (τ) signal in the range of 1 to 999 x 10^6 μsec. The dead time between measurements (τ_d) is also controllable via the plug-in. The plug-in also stores the measurement data in a buffer memory for output over the interface bus to the calculator.

The measurement cycle of the counter is also controlled by the plug in by-passing such things as the display cycle to minimize the dead time between measurements. Dead time as low as 15 μs can be achieved.

The mixer/amplifier unit provides a means to further increase the system's resolution by heterodyning the test signal down to a low frequency signal by mixing it with an offset reference oscillator. It includes the necessary filtering, bandwidth control amd amplifiers to properly condition the input signals for application to the counter.

The control of all the previously described instruments is provided by the calculator via the interface bus. Measurement data is also sent to the calculator by the same means. The calculator is programmed from its keyboard to perform the various measurement operations and to process the data received from the counter and plug-in. All the aspects of system behavior are under program control of the calculator. The operator specifies measurement parameters at the keyboard. The printer device is used to output the processed results either in numeric or plotted form.

System Operation

For the following discussion,refer to figures 7 and 8.

Signal Conditioning. There are a variety of ways of preconditioning the input signal before applying them to the input of the counter. The major objective of these techniques is to increase the resolution of the measurement. The one shown in Figure 8 is a simple heterodyning technique where the test oscillator is compared with a reference oscillator which is offset in frequency from the test oscillator by ν_i. The output of the two oscillators is mixed together to produce an audio range beat note. The output of the mixer is passed thru appropriate filtering to eliminate the undesired mixer products. The signal is then amplified by a high gain, low noise, limiting amplifier. The main purpose of the amplifier is to provide reliable detection of the zero crossings of the beat note. The output of the amplifier is essentially a square wave which is used to drive the input of the counter. This approach is necessitated by the wide bandwidth (500 MHz) of the counter input and the resulting input noise which makes it impossible to detect a low frequency zero crossing with a 2 nsec resolution. As such the amplifier is provided with bandwidth control so that the input noise bandwidth can be adjusted to be compatible with the signal being measured.

The technique of comparing two oscillators, while it has some disadvantages, is still the best overall method of performing this measurement. It is not possible to measure the carrier directly, especially at microwave frequencies, with sufficient resolution, thus some down conversion technique is required. The problem is the noise contributed by the local (reference) oscillator. In order for it to be negligible its spectra must be 10 to 20 dB below the test oscillator in which case the error is small (.4 to .2 dB respectively). When it is not possible to obtain a reference oscillator which is better than the test oscillator, then two oscillators of assumed identical spectral characteristics can be used and the measurement result taken as the average of the two. This approach will always measure the worst of the two with a maximum error of less than 3 dB. The uncertainty of which oscillator is the poorest of the pair can be resolved by taking 3 or more and comparing them in all combinations (A vs B, A vs C, B vs C, etc.). This approach will identify both the best and the worst of the group.

The remaining problem with the heterodyning technique is the requirement that the reference oscillator be offset from the test oscillator. In certain cases (cesium, rubidium or crystal oscillators for example), it is difficult or impractical to offset them, and as such, a different signal conditioning approach is required. These have been discussed in the literature[8]. The main requirement in terms of the current approach is to present the counter with a signal which is representative of the phase of the test oscillator and whose period is measurable by the counter with sufficient resolution to be meaningful.

Counter Operation. The counter counts the number of cycles of the input signal that occurs within the gate time and the number of cycles of the 500 MHz time base that occurred from the first zero crossing after the opening of the counter's main gate to the last zero crossing after the closing of the gate. Thus, the measurement consists of two numbers: 1) the number of cycles of the input and, 2) the elapsed time in 2 nsec steps that it took the n cycles to occur.

The operation of the counter and plug-in unit are directed by commands received over the interface bus from the calculator. These are determined by the stored program in the calculator. Each series of measurement is made by programming the counter to the desired function (period, frequency, time interval), setting up the desired gating function (measurement and dead time) and the number of measurements to be made. The resulting data is stored in the plug-in in order to reduce the dead time between measurements and subsequently transferred to the calculator via the interface bus for processing.

Sampling Functions.

The frequency selective characteristics of the counter is determined by the way the measurement data is acquired by the counter (i.e. the measurement and dead time) and the processing algorithm used in the calculator (i.e. the choice of α's) as given by eq. 24. A variety of resulting transfer functions have been discussed in the literature[6,7]. The most attractive of these from the point of view of an on line process is the so called modified Hadamard variance or 50% dead time sampling function proposed by Baugh (see Reference 7, p 225, Figure 6). The reason for this choice is the sampling function has two α coefficients which are zero and the counter can be allowed to reset and transfer data during these intervals. Hence the counter is essentially free from dead time constraints when using this sampling function. Secondly, the sampling function is a short sequence suitable to on line processing & allows measurements to be made out to a reasonable distance away from the carrier.

Software.

The software (the stored program in the calculator) determines a significant portion of the system's behavior and performance characteristics. The set-up of the operating modes of the instrument and the method of data reaction is determined by the program written for the calculator. As such the system has a great deal of flexibility in executing various sampling and processing methods as a function of writing the appropriate program.

System Performance

Sensitivity: The system's sensitivity using the simple heterodyning method and the 50% dead time sampling function is given by eq. 43. Evaluating this gives a family of curves as shown in figure 9. Eq. 43 assumes no other sources of noise in the system. This is valid as long as the mixer and amplifier noise are designed to be below this limit. As can be seen, the sensitivity increases with a decrease in the beat frequency (ν_i) and as the offset frequency is increased.

In the primary region of interest (10 Hz and below), the sensitivity is quite good compared to most oscillators available today. In the region above 10 Hz, which is primarily for comparison with other techniques, the sensitivity can be inadequate. In these cases other resolution enhancement techniques, such as multiplying the input signals to microwave frequencies, can be used.

Maximum Offset Frequency: The maximum offset frequency is limited by the counter's dead time and number of measurements required per cycle of the sampling function. In the case of the 50% dead time sampling, the upper offset frequency is given by:

$$f_{max} = \frac{1}{6\tau_d}$$

Since the counter is only able to measure an integral number of cycles of the input, one of these is equal to the dead time, this can also be related to the beat note frequency, thus

$$f_{max} = \frac{\nu_i}{6}$$

Filter Bandwidth: The approximate equivalent filter bandwidth as given in figure 5 is

$$B = \frac{f_o}{N}$$

It is important to note two characteristics: 1) As the filter fundamental response f_0 is moved closer to the carrier, the bandwidth of the filter becomes proportionally smaller, and 2) By increasing N, the bandwidth can be made arbitrarily small. Both of these facts are what allows this method to measure phase noise arbitrarily (in theory) close to the carrier, whereas traditional analog methods are limited by the skirts of analog filters.

Digital Filter Harmonic Responses

As shown in figure 6, the harmonics of the digital filter have the same response as the fundamental response. The usual assumption is that the spectra in the region of interest is declining at a rate of f^{-2} or greater, and that the number of harmonic responses is limited to a finite number by the selectable IF filter and thus the error is negligible. However, this assumption must be verified each time a different class of oscillator is measured as white phase noise can contribute significantly. Further, so called bright lines occurring at any of the harmonic responses will produce erroneous results. The problem of white phase noise is discussed by Lesage and Audoin[6]. The problem of bright lines (usually 60 and 120 Hz and harmonics thereof) can be coped with by judicious choice of the offset frequencies and/or bandwidths.

Performance Verification

Several methods were used to verify the results obtained by this measurement technique.

FM Modulation: By applying a FM modulated signal of a known index of modulation and using the relationship given by eq. 40, the measured value of the sidebands can be compared with those values predicted by the modulation index.

The index of modulation is given by

$$\beta = \frac{\Delta f}{f_m} = \theta_p$$

where f_m is the modulation frequency. If the modulation index is small, $\beta << \frac{\pi}{2}$, then the value of the sideband at $f_c + f_m = \beta/2$.

Sensitivity Floor Verification: The system's resolution limit as given by eq. 43 was verified experimentally. A test tone was generated by dividing a good crystal oscillator's output frequency down to a variety of frequencies suitable as a beat note input to the system. The frequency dividers were digital circuits (74LS161's) operated in a manner so as to introduce a minimum of phase noise. The resultant signals could be calculated to be much below the sensitivity limit of the system. If the square wave outputs of the dividers were applied to the amplifier, the results were much better than the model predicted. This is explained by the fact that the square wave eliminated trigger errors and both the counter's time base and the test tone were derived from very stable crystal oscillators and a degree of synchronization occurred. If the outputs of the dividers were filtered to create sine waves, a ± one count error due to trigger noise could be observed and good correlation to the resolution floor model occurred.

Correlation with Other Techniques: In the region of offset frequencies where other techniques can be used, measurements were made on various oscillators and the results compared. Results correlated within 3 to 6 dB. One difficulty with correlating these results is estimating the mean value of the spectra using spectrum or wave analyzers. This tends to be subject to operator judgment.

Sample Results

A typical printout of the system is shown in figure 10. A series of measurements are made at the specified frequencies and digital filter bandwidths. The measurements are repeated at each frequency for specified numbers of "sweeps" to provide some statistical information about the measurements since the measured value represents an estimation of the mean value of the spectra. The results can also be presented in graphical form as shown in figure 11. Here the "X" represents the average of the values measured and the "-", the one sigma value of the variations.

Conclusions

The use of time domain techniques for close-in phase noise measurements can now be performed in a practical manner. Use of a high resolution reciprocal counter gives good sensitivity for offset frequencies below 100 Hz. By combining a programmable calculator with the programmable instrumentation, results can be obtained in the time it takes to collect the data.

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FIGURE 6

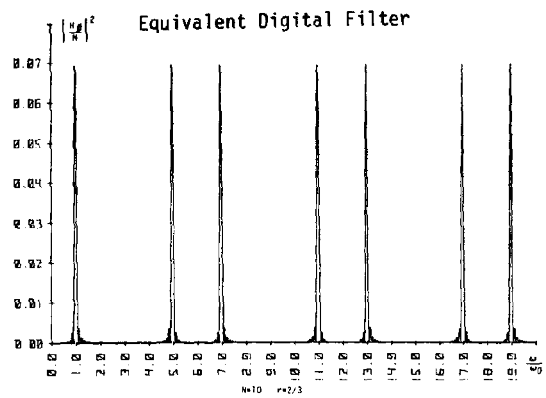

Figure 7 HP 5390A Frequency Stability Analyzer System Configuration

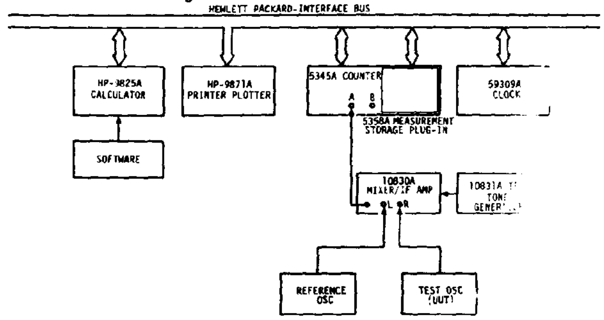

FIGURE 8

Phase Noise Measuring System Functional Block Diagram

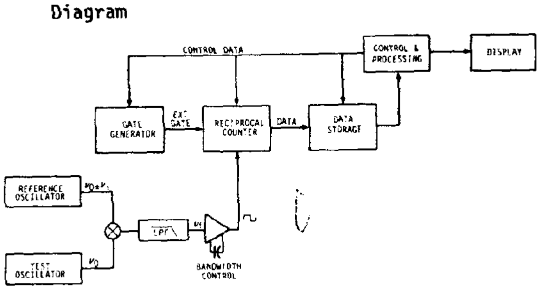

FIGURE 9

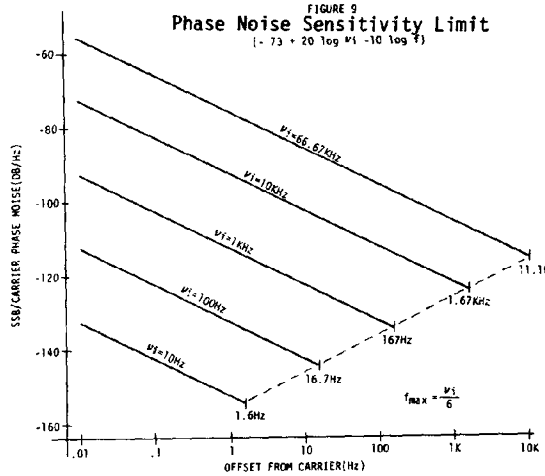

FIGURE 10

Sample Printout

*** P H A S E   N O I S E   A N A L Y S I S ***

PHASE NOISE PROGRAM (5625) rev 05-26-76-2310

MEASUREMENT DESCRIPTION:

10544A-L36 VS 10544A-H36
DATE 05/26/76 TIME 23:23:00

MEASUREMENT PARAMETERS:

K= 10
IF FREQUENCY 1000 HZ

SSB/CARRIER PHASE NOISE (DB/HZ)

FREQ=	10.4	2.5	1.0	0.5	0.2	HZ
BW =	0.52	0.12	0.05	0.03	0.01	HZ
FLOOR	-123.2	-117.0	-113.0	-110.0	-106.0	DB
SWEEP						
1	-125.1	-116.1	-103.5	-91.8	-79.9	
2	-126.0	-114.4	-104.6	-95.2	-80.8	
3	-125.8	-112.7	-103.7	-93.6	-79.7	
4	-124.0	-117.7	-104.7	-91.5	-77.7	
5	-122.3	-118.9	-103.6	-92.3	-77.2	
AVE=	-124.4	-115.4	-103.9	-92.7	-78.8	
SIG=	1.5	2.5	0.6	1.5	1.6	
MIN=	-126.0	-118.9	-104.7	-95.2	-80.8	
MAX=	-122.3	-112.7	-103.0	-91.5	-77.2	

END TIME:01:52:03

FIGURE 11