2 Getting Off the Ground

Building on sequential small successes always makes the journey more fun in my estimation. The first tasks that I undertook in my telescope making ventures are described in this section.

2.1 Spherometer for Radius of Curvature Measurement

It is very nice to be able to quickly estimate the radius of curvature of a spherical surface without having to set the subject lens or mirror up in a separate stand to do testing. This is particularly true during the rough grinding stages when the surface reflectivity is typically quite poor. A good spherometer fills this role nicely.

The spherometer makes use of the simple relationship between a circle (sphere) and the sag of that circle between points separated by a known distance as shown in Figure 4. A simple application of the Pythagorean Theorem makes it possible to write the mirror sag as

$$\delta z = R - \sqrt{R^2 - h^2} \tag{1}$$

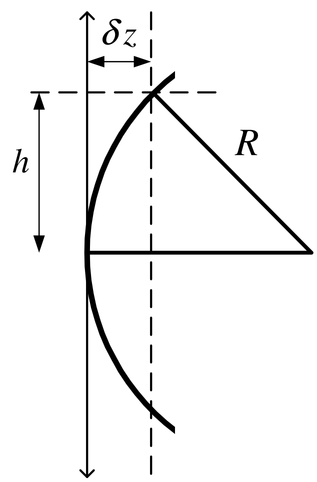

Figure 4 Simple diagram relating the sag of a spherical surface δz, the height above the optical axis h, and the radius of curvature R (equal to twice the paraxial focal length)

It is useful to use (1) to estimate how accurately we will be able to estimate the radius of curvature for different mirror diameters and f-numbers. A few cases are collected in Table 1. As shown there, this configuration should allow very accurate estimates to be made for the surface's radius of curvature.

Table 1 Spherometer Measurement Domain

Mirror Diameter, in	F-Number	R, in	h, in	δz, in	Resolution[1] as % of Sag
6	5	60	2	0.033343	0.15
10	4	80	2	0.025000	0.20
			4	0.100060	0.50
12	4	96	2	0.020840	0.24
			4	0.083370	0.06
	2.5	60	2	0.033343	0.15

[1] Assumes dial resolution of 0.00005".

I personally prefer to use a spherometer that has "ball feet" as shown in Figure 5 because this should help alleviate the possibility of scratching a mirror's pristine surface. This does, however, complicate solving for the mirror sag (δz in Figure 4) because the separation between the two points at which the feet touch the mirror surface is a function of the ball diameter r and the angular quantity ϕ as shown. Consequently, the distance between the bottom of the horizontal plate and the lowest point of the mirror's arc H is also a function of these quantities. An additional source of measurement error comes into play if the dial indicator (not shown in Figure 5) is positioned δh away from the midway point between the two ball feet resulting in the measurement *H'* being made rather than *H*.

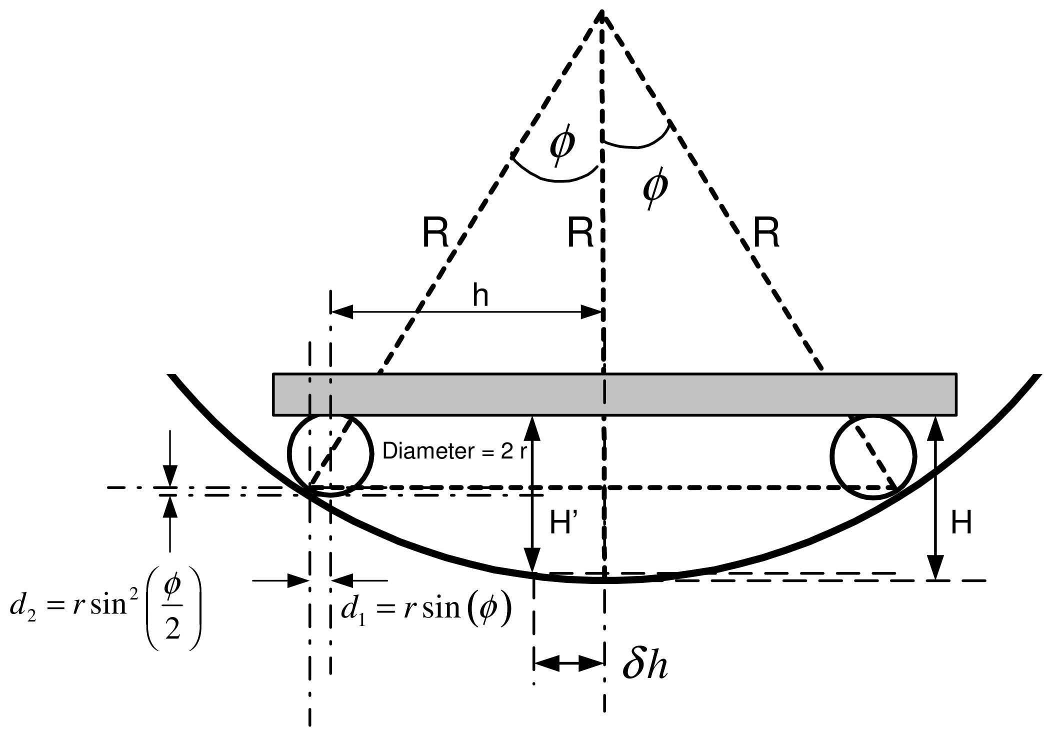

Figure 5 Geometry associated with ball-footed spherometer for error analysis

Similar triangles can be used to recognize that

$$d_1 = r\sin(\phi) \tag{2}$$

Quantity d_2 is given by

$$\begin{aligned} d_2 &= r - \sqrt{r^2 - d_1^2} \\ &= r - \sqrt{r^2 - r^2\sin^2(\phi)} \\ &= 2r\frac{1-\cos(\phi)}{2} \\ &= 2r\sin^2\left(\frac{\phi}{2}\right) \end{aligned} \tag{3}$$

Although d_2 is much smaller than d_1 because of the squaring, we care about being as precise as possible here because this term obviously affects *H* and *H'* in a direct way. Take for instance one example; a 6-inch diameter mirror with an f-number of 4. In this case, R = 48 inches. Similar triangles in Figure 5 make it possible to write

$$\phi = \sin^{-1}\left(\frac{h}{R-r}\right) \tag{4}$$

If h = 1 inch and r = 0.125 inch, this leads to ϕ = 1.19687°, d_1 = 0.00261 inch, and d_2 = 0.0000273 inch which is appreciable (equal to about one-half of the dial indicator's resolution that I will be using). Therefore, these minute details are still important in achieving good measurement accuracy.

As alluded to earlier, if the measurement point is not centered exactly between the two ball feet, the measured mirror sag will be in error by $H - H'$ because $\delta h \neq 0$. For $\delta h = 0$, however,

$$H = 2r - d_2 + R - \sqrt{R^2 - (h + d_1)^2} \tag{5}$$

From (2) and (4), it is clear that

$$d_1 = \frac{rh}{R - r} \tag{6}$$

The quantity d_2 is a bit more complicated, but from (3) and (4) we can write

$$d_2 = r\left[1 - \sqrt{1 - \left(\frac{h}{R - r}\right)^2}\right] \tag{7}$$

Using these last two results in (5) produces

$$H = 2r - r\left[1 - \sqrt{1 - \left(\frac{h}{R - r}\right)^2}\right] + R\left[1 - \sqrt{1 - \left(\frac{h}{R}\right)^2\left(1 + \frac{r}{R - r}\right)^2}\right] \tag{8}$$

If the depth gauge is not centered exactly halfway between the two ball feet in Figure 5, the depth reading will be in error by δH where

$$\delta H = R - \sqrt{R^2 - (\delta h)^2} \tag{9}$$

and of course from the same diagram, $H' = H - \delta H$.

Given the measurement H', a closed-form solution for R is fairly involved. It is easier to simply use numerical methods (e.g., Newton-Raphson) to find the solution iteratively. Solutions for h = 1" and h = 2" are given in Figure 6 and Figure 7 respectively. The measurement error due to the alignment error δh using (9) is shown graphically in Figure 8.

My first attempt at assembling a spherometer is shown in Figure 9 and Figure 10 using a high-quality digital electronic indicator dial gauge (0.5" / 0.00005") from Anytime Tools. Aside from its excellent resolution and accuracy, this gauge can be conveniently zeroed when calibrating to a reference flat surface and this makes subsequent readings very convenient. I married this gauge with simple hardware from Ace Hardware just to get a feel for the precision involved along with other details. If you are used to working primarily with metal rather than wood, the precision required will probably surprise you. The aluminum plate should ideally be a minimum of 0.25" thick in order to be adequately rigid. If standard bolts are ultimately used for the supporting feet, the increased plate thickness will also help insure that the bolts are perfectly vertical. The additional weight of a thicker plate also helps to prevent the spherometer from being front-heavy and potentially falling over.

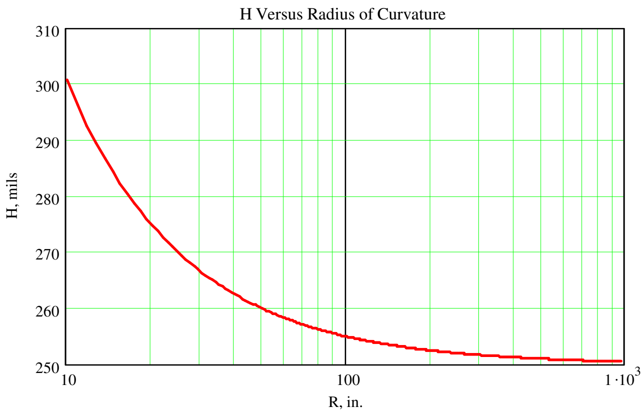

Figure 6 H versus[2] R for h = 1" and r = 0.125"

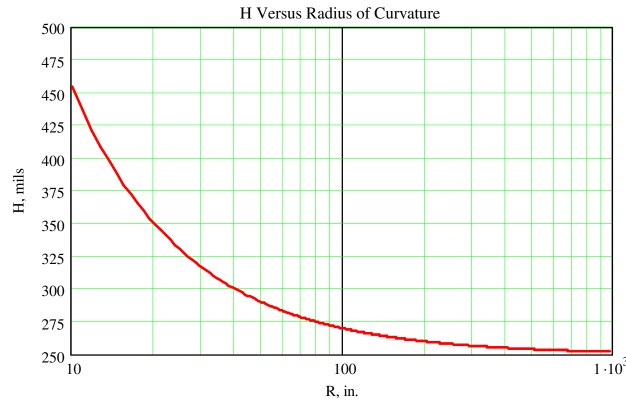

Figure 7 H versus R for h = 2" and r = 0.125"

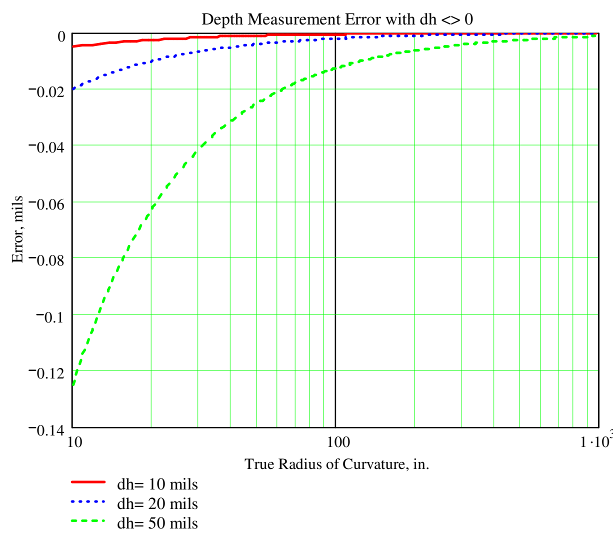

Figure 8 Sag measurement error[3] due to $\delta h > 0$ in (9)

Figure 9 My first spherometer attempt shown atop my old 6" mirror blank from high school days

Figure 10 Edge-on view of the spherometer more clearly showing the mirror's sag. The supporting T-plate is so thin in this example that the make-shift bolt-feet can be easily misaligned.

[2] Computed using U15620 Spherometer Calculations with Ball Feet.mcd.
[3] Ibid.

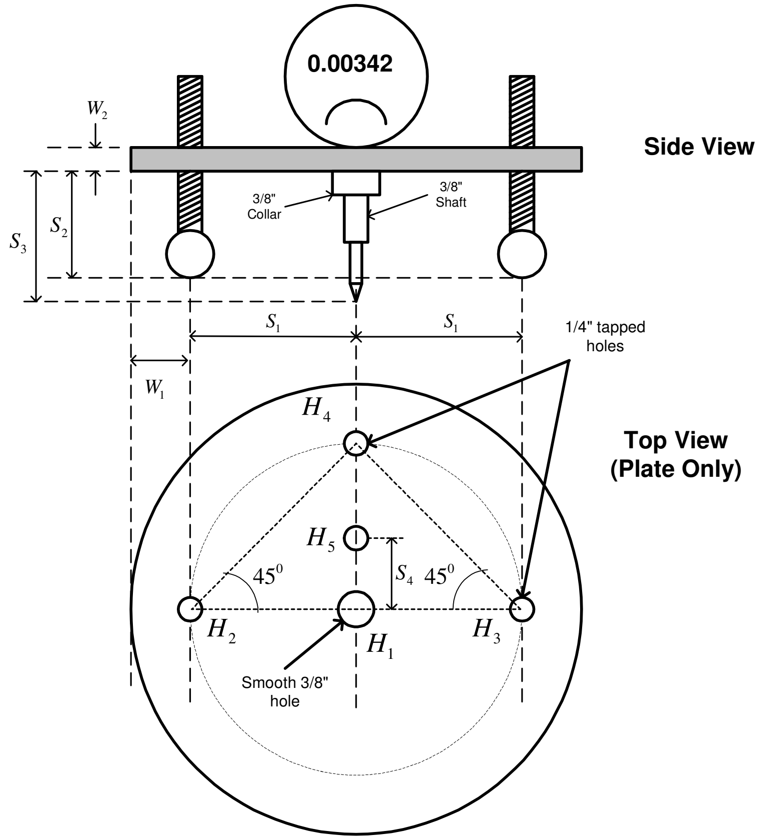

Figure 11 Final mechanical design for spherometer using a solid circular plate for good support

Table 2 Mechanical Details for Figure 11

Parameter	Detail	Comments
W_1	0.500 in.	
W_2	0.375 in.	Uniform aluminum plate thickness
S_1	1.500 in.	Center-to-center hole spacing
S_2	1.35 in.	Nominal
S_3	1.60 in.	With gauge pin fully extended
S_4	0.67 in.	Center-to-center spacing from main gauge pin to center of slot.
H_1	3/8”	Smooth bore hole for precision mechanical gauge
H_2, H_3, H_4	0.25 in / 20 threads/in. (20 UNC)	All the way through the plate.

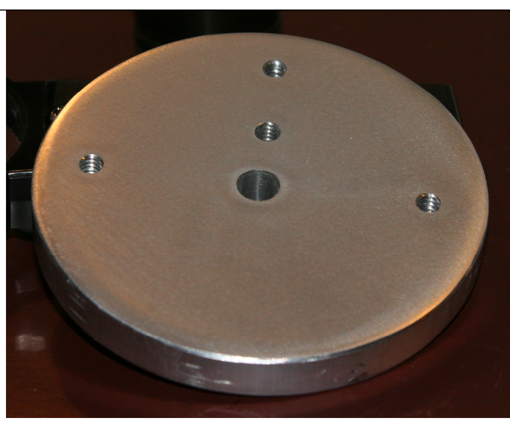

Figure 12 Aluminum plate with holes drilled, tapped, and surface smoothed

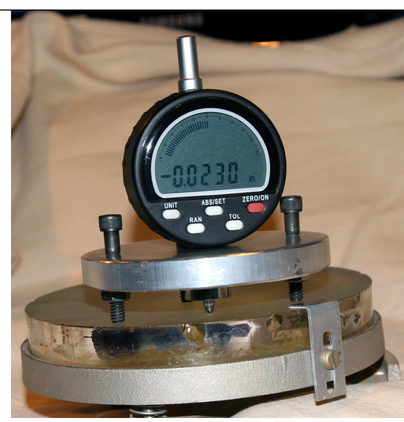

Figure 13 Spherometer less ball feet atop a 6” mirror blank

Figure 14 Under-side of the spherometer

Figure 15 View of completed spherometer with glass ball feet now attached

2.1.1 Further Improvements

The most obvious improvement that is needed to the design is that the present design falls over meter face first because of the symmetry and center of gravity associated with Figure 11. This can be easily remedied with a small amount of weight placed around the back cap bolt. Alternatively, the design in Figure 11 can be slightly modified to that shown here in Figure 16 with the precise details given in Table 3. This modification moves the center of gravity such that the spherometer remains normally upright.

The region of the plate in the vicinity of the 3/8" collar shown in Figure 14 should ideally be milled out so that more of the gauge's center column fits through the collar. At present, only a small portion is available for the collar's set-screw to contact. This improvement will necessarily have to wait until I have access to a milling machine.

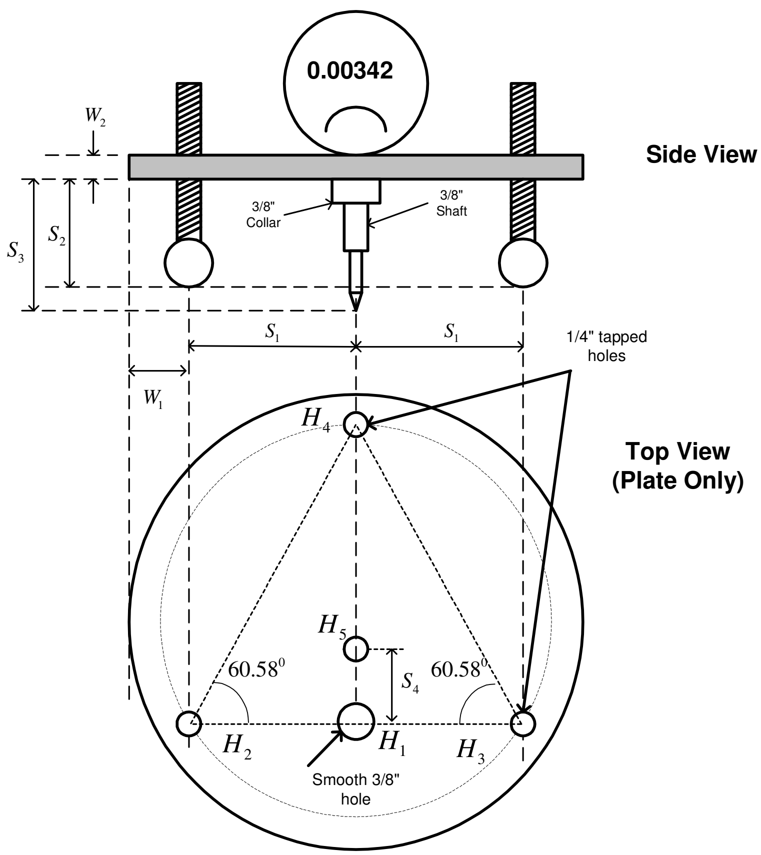

Figure 16 Alternative dimensioning for the spherometer that moves the center of gravity such that the meter does not fall over. The distance between the H_2 and H_3 holes is the same as used in Figure 11.

Table 3 Mechanical Details for Figure 16

Parameter	Detail	Comments
W_1	0.500 in.	
W_2	0.375 in.	Uniform aluminum plate thickness
S_1	1.500 in.	Center-to-center hole spacing
S_2	1.35 in.	Nominal
S_3	1.60 in.	With gauge pin fully extended
S_4	0.67 in.	Center-to-center spacing from main gauge pin to center of slot.
H_1	3/8"	Smooth bore hole for precision mechanical gauge. Center of hole is on centerline situated 0.91" from the center of the 4" diameter disk.
H_2, H_3, H_4	0.25 in / 20 threads/in. (20 UNC)	All the way through the plate. Dimensions in inches. Center of H_2 located at (–1.5, –0.91). Center of H_3 located at (1.5, –0.91). Center of H_4 located at (0, 1.75).

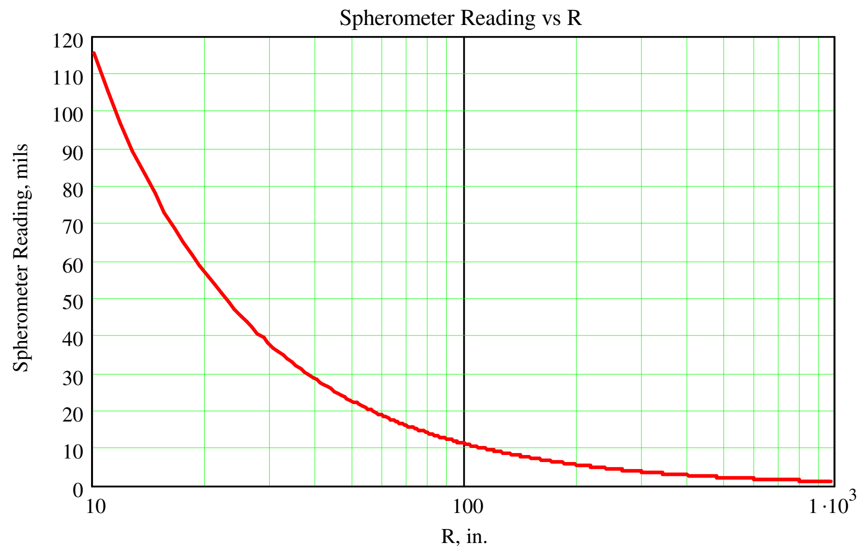

Figure 17 Spherometer reading[4] versus radius of curvature for h = 1.5" and 0.40" diameter ball feet

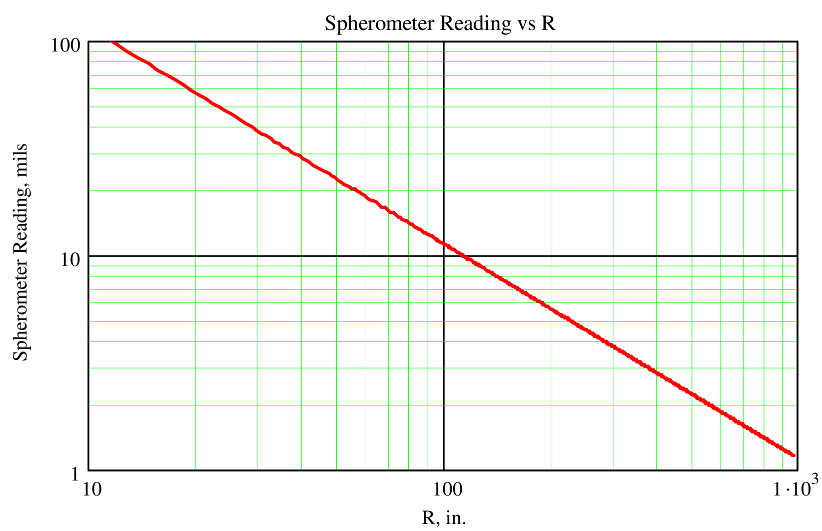

Figure 18 Alternative log-log plot of the same information presented in Figure 17

[4] From u15620 Spherometer Calculations with Ball Feet2.mcd.

