

Select mixer frequencies painlessly. With a simple, graphical method, you can sidestep long trial-and-error sessions and determine bandwidth, too.

If you've ever become bogged down in the calculation of mixer frequencies, you'll appreciate a simple, graphical selection technique that avoids the voluminous printouts of computer aided iterations and trial solutions.

With this procedure, you can visually select the appropriate mixer frequencies to meet your spurious requirements. Only those spurs whose levels exceed the given spec are plotted on the chart. As an added benefit, you can map the output bandwidth onto the spur chart as a function of the bandwidths of the two input signals. The region thus described is an irregular hexagon with sides defined by the relative bandwidths and ratios of the mixing tones.

This definition of the locus of output frequencies mapped on the spur-chart plane lets you select mixer frequencies rapidly and lends further usefulness to the chart. (The spur chart has been available since 1966 but has not been widely used because of a poor understanding of bandwidth effects.)¹

Derivation of equations

Typical spurs of high and low-level mixers are shown in Figs. 1 to 4. The ordinate is the frequency ratio f_1/f_2 , and the abscissa is the percentage bandwidth with respect to the desired output frequency, f_o , where $f_o = f_1 + f_2$ for the sum charts, and $f_o = f_2 - f_1$ for the difference charts ($f_2 > f_1$).

To derive the equations that generate the graphs, note that the spurs, or cross-modulation products, have the general form

$$P = Mf_1 + Nf_2 \quad (1)$$

where M and N are positive or negative integers. The frequency ratio, $n = f_1/f_2$, is always less than 1. Percentage separation is given by:

$$S = \frac{P - f_o}{f_o} \times 100 \quad (2)$$

$$\text{or } P = \frac{f_o S}{100} + f_o \quad (3)$$

Eq. 1 and 2 can be rewritten for the case $f_o = f_1 + f_2$:

$$f_2 = \frac{P - Mf_o}{N - M} \quad (4)$$

Then the frequency ratio, n, can be expressed, with use of Eq. 4, as

$$n = \frac{f_1}{f_2} = \frac{f_o - f_2}{f_2} = \frac{Nf_o - P}{P - Mf_o} \quad (5)$$

Eq. 4 and 5 are both undefined for the case $N = M$. To simplify calculations, such spurs are deleted from this computation. It can be shown, however, that the $N = M$ spur lines are vertical and appear on the summing mixer chart at $S = (N - 1) \times 100$. Similarly the analysis of difference mixing does not consider spurs where $N = -M$. These, too, are vertical lines on the chart, with $S = (N - 1) \times 100$.

Now, with use of Eq. 3, Eq. 5 becomes

$$n = \frac{-S + 100(N - 1)}{S - 100(M - 1)} \quad (6)$$

This equation relates n, the frequency ratio, to S, the percentage separation, for $f_o = f_1 + f_2$, and is used to plot the sum charts.

Similarly, for $f_o = f_2 - f_1$, Eq. 1 can be rewritten

$$f_2 = \frac{P + Mf_o}{N + M} \quad (7)$$

Again, the frequency ratio, n, can be expressed, with use of Eq. 7, as

$$n = \frac{f_1}{f_2} = \frac{f_2 - f_o}{f_2} = \frac{P - Nf_o}{P + Mf_o} \quad (8)$$

Now, using Eq. 3, we see that Eq. 8 becomes

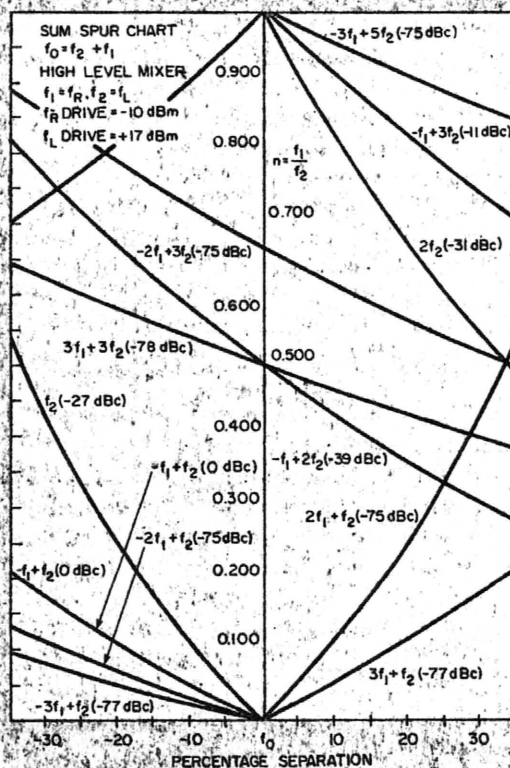
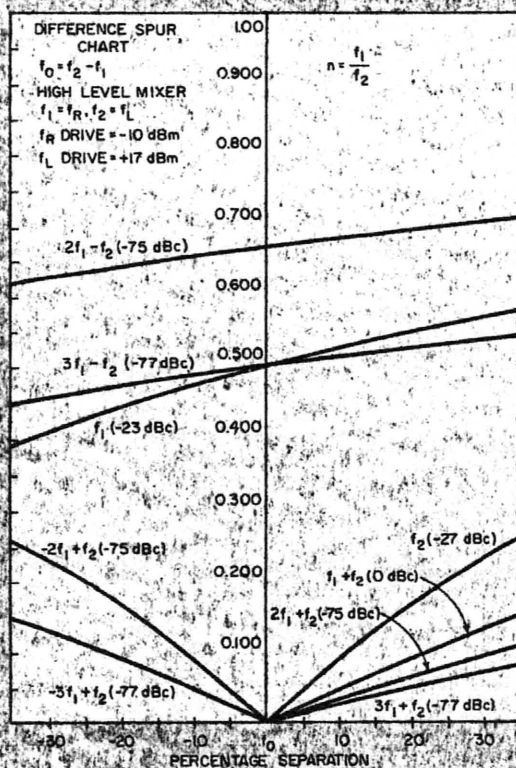
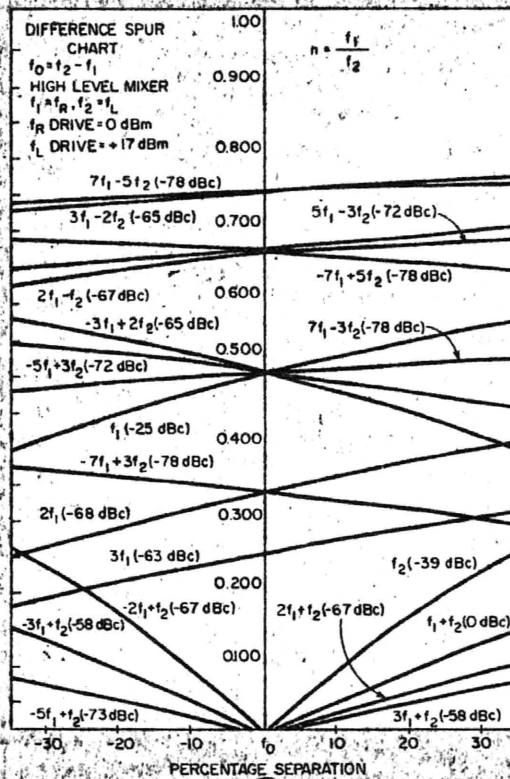
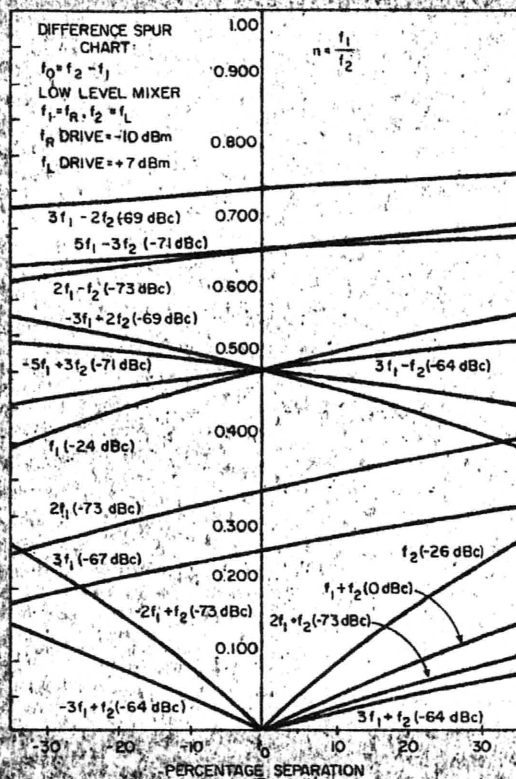
$$n = \frac{S + 100(1 - N)}{S + 100(1 + M)} \quad (9)$$

Eq. 9 relates n and S for $f_o = f_2 - f_1$, and is used to plot the difference charts.

Calculator does the plotting

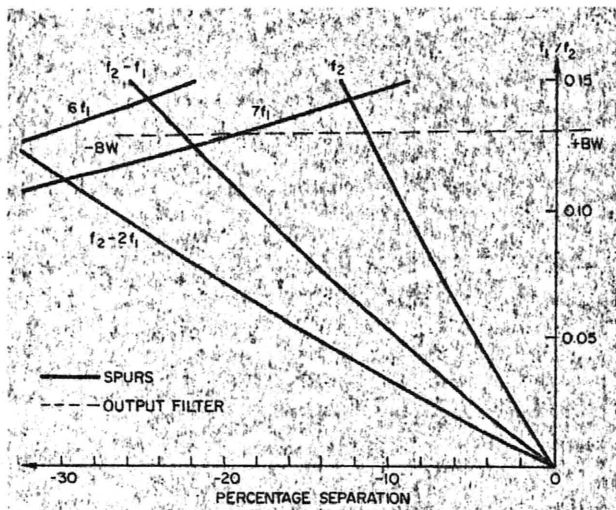
A program written for the HP 9810 calculator uses Eqs. 6 and 9 to plot spurs for any M and N. If all the spurs to the seventh order are plotted, the graphs are seen to be identical to those in reference 1. However, many of the spurs that are plotted with this procedure are very small in

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1-4. Harmonic spurs of mixers can be charted as a function of output bandwidth for differences (1 to 3) and sums (4). Note that about the same spurs appear in

both low and high-level mixers (1 and 2). When signal drive is decreased in the high-level mixer, however, the spurs are reduced (3).



5. Filter output-band mapping is demonstrated with a section of the summing-mixer spur chart. Harmonics are easily located relative to the output bandwidth.

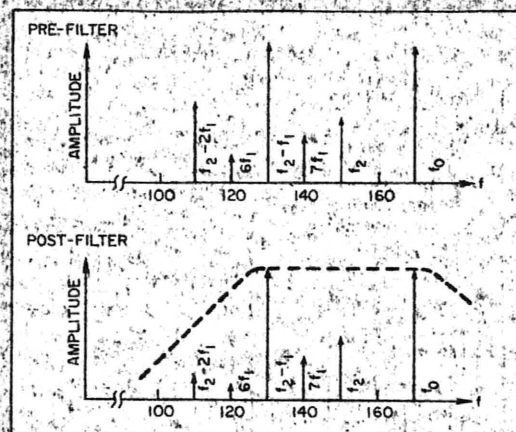
magnitude. The advantage of the program is that spurs can be selectively plotted, that is, those spurs smaller than a given level can be omitted; only those large enough to cause concern need be plotted.

The levels of the spurs for the various orders listed are typical of low-frequency (50 MHz) mixers and are found in the Watkins-Johnson mixer catalog. One caution: Don't use these charts to design a receiver without first measuring spur levels for the actual mixer under approximate operating conditions.

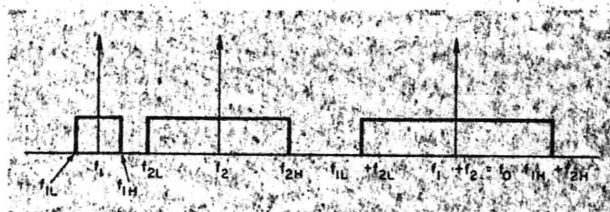
A difference-mixer spur chart for a low-level mixer (WJ M1) with signal drive of -10 dBm and a local-oscillator drive of $+7$ dBm appears in Fig. 1. Fig. 2 is a difference-mixer spur chart for a high-level mixer (WJ M1D) with signal drive of 0 dBm and a local-oscillator drive of $+17$ dBm. Note that essentially the same spurs are present in both cases. The same mixer is depicted in Fig. 3, but with signal drive decreased to -10 dBm. Note the reduced spurs. Clearly, a high-level mixer with low signal drive is the best choice here to minimize spurious frequencies.

The summing-mixer spur chart for the mixer and conditions of Fig. 3 is given in Fig. 4. As can be seen, the summing-mixer chart is more cluttered than that of the difference mixer, but it still offers many promising regions for up-conversion. Note that this technique considers only harmonic spurs in the mixer and not two-tone products.

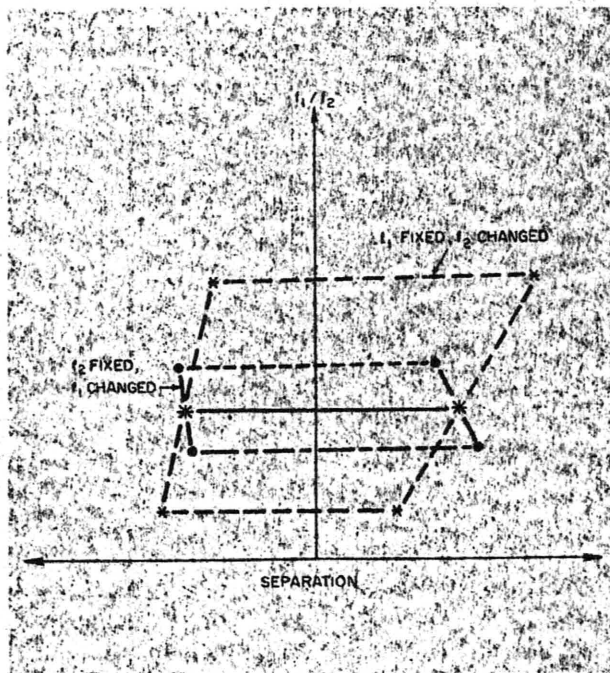
These charts are used for a single-frequency local oscillator and a single-frequency signal with modulation. The frequency ratio must be chosen so that no spur is found within the bandwidth of the modulation. However, a mixer is normally used not just for circuits that combine fixed fre-



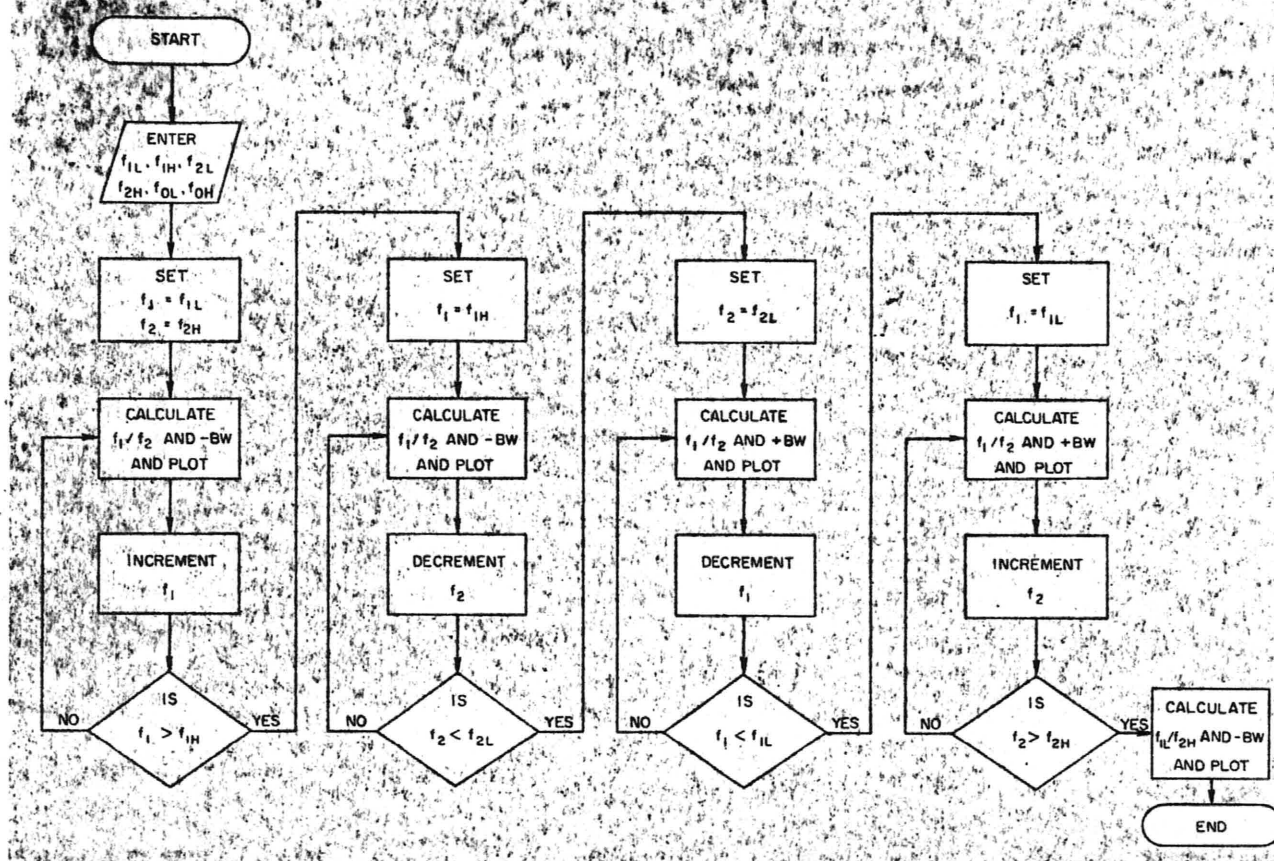
6. Output spectrum for pre-filtering (top) and post-filtering (bottom) shows that filtering reduces the out-of-band spurs but not the in-band harmonics.



7. Upconversion of two frequency bands: frequencies f_1 and f_2 lie at the center of each of the respective bands. Output band appears at right.



8. The upconverted output of Fig. 7 appears on the spur-chart plane distributed about the f_1/f_2 point. Incremental effects of frequency changes are easily spotted.



9. Calculator program plots the hexagonal locus of points resulting from variations in the values of f_1 and f_2 . The dimensions of the hexagon vary with the ratio of the

input bandwidths: For zero bandwidth, the hexagon becomes a quadrangle. The hexagon can be plotted onto the spur chart with an HP 9810 calculator.

frequencies but for those that mix one frequency band with another—as in a synthesizer or receiver design. To design this type of system, you must map the output filter onto the coordinate system of the spur chart to produce an easily read graphic representation of harmonic spur location relative to the output bandwidth.

How to use the charts

To accomplish such a mapping, consider first the simple case of mixing two fixed frequencies. Given the two input frequencies and an output frequency range, you can plot a line for the function of percentage separation vs f_1/f_2 . (The value of f_1/f_2 corresponds to the ratio of signals producing the selected output tone $f_1 + f_2$ or $f_2 - f_1$.) The end points of the line can be called $-BW$ and $+BW$ and are calculated from Eq. 2:

$$+BW = \frac{f_{BW} - f_0}{f_0} \times 100, \quad (10)$$

$$-BW = \frac{f_{BW} - f_0}{f_0} \times 100 \quad (11)$$

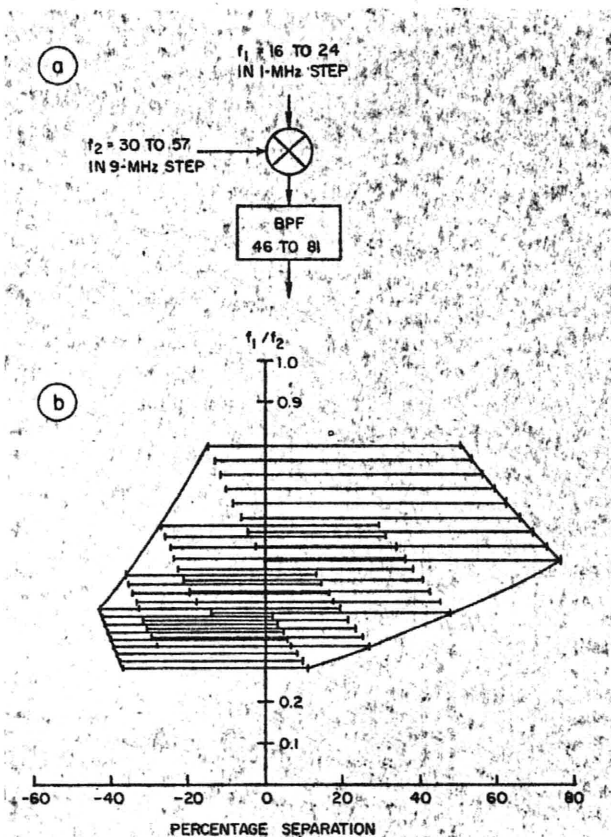
where f_{BW} is the upper edge of the output filter

passband, f_{BW} is the lower edge, and f_0 is the desired instantaneous output frequency.

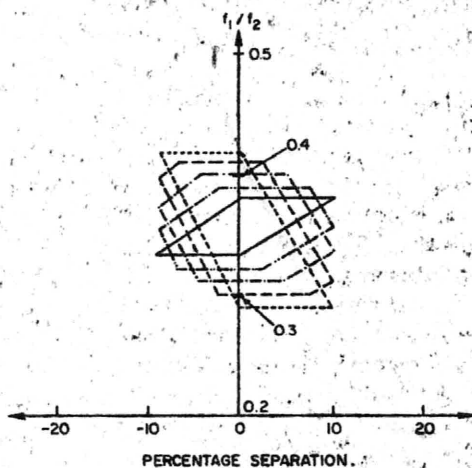
The points along this straight line correspond to frequencies within the filter passband. The curves for various orders of harmonic spurs are displayed similarly—that is, each point on the line corresponds to a frequency. (This frequency is determined by the specific values of f_1 and f_2 , thus defining a specific point on the ordinate.)

The use of these graphic definitions can best be explained with an example. A small portion of the summing-mixer spur chart may look as shown in Fig. 5. Let $f_1 = 20$ MHz and $f_2 = 150$ MHz. (This will quickly prove to be a poor frequency assignment.) Then $f_1/f_2 = 0.133$. Also, define the output filter bandwidth to span 125 to 175 MHz. The spur chart indicates that for this f_1, f_2 value, the filter bandwidth extends from 3% above f_0 to 26% below f_0 . The chart also shows the f_2 , $7f_1$, and $f_2 - f_1$ spurs intersecting the filter line. These points indicate in-band harmonics, as can be verified numerically.

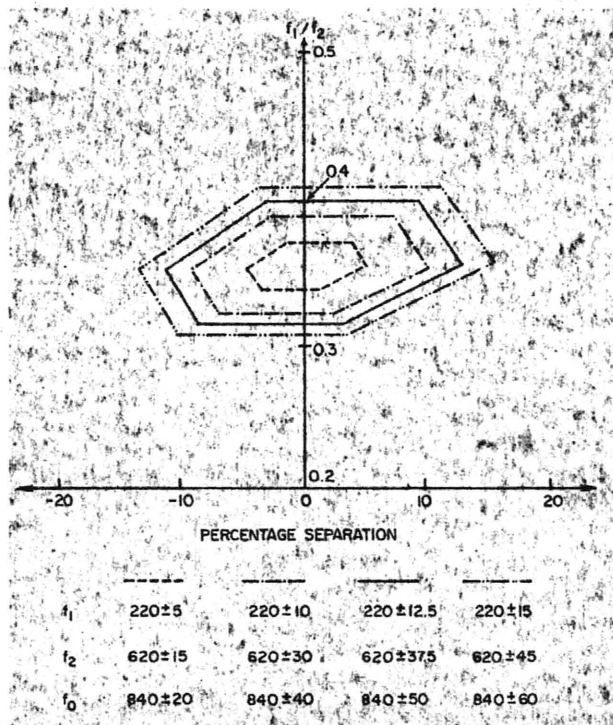
Plots of the output spectrum (prefiltering and postfiltering) for this example are shown in Fig. 6. The in-band spurs—those intersecting the filter



10. Calculator program for the circuit in (a) maps a 'hexagonal' perimeter when f_1 and f_2 vary in infinitesimal steps (b).



11. Design example: Figures generated by five different bandwidths show the advantage of a small bw at f_1 .



12. Other trends are shown by various frequency assignments. Shown are the benefits of a smaller bw for f_1 .

line on the spur chart—are not reduced by filtering and will be present at whatever amplitude the mixer propagates. Out-of-band spurs will be attenuated.

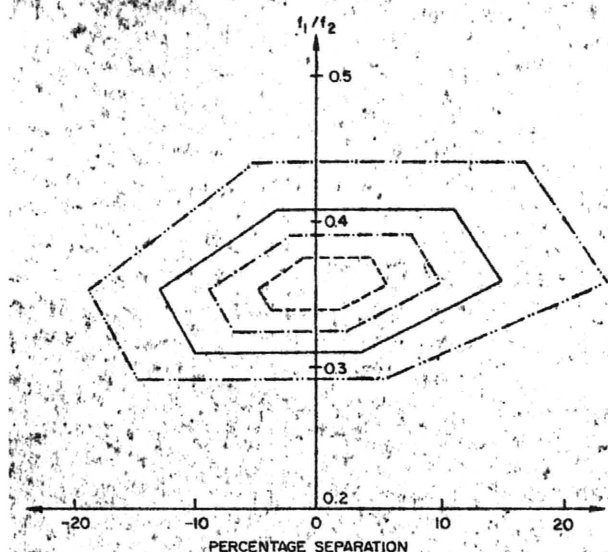
In fact, since the horizontal scale of the spur chart is a measure of frequency, you can use the horizontal distance from the filter band edge to a spur line to estimate the position of the spur in the filter skirt and thus its attenuation.

Consider, then, upconversion of two frequency bands as shown in Fig. 7, and assume that the bandwidth of f_1 is less than that of f_2 . If you fix f_1 and f_2 at the center of their respective bands, the output appears on the spur-chart plane, as shown by the solid line in Fig. 8: The bandwidth is equally distributed about the f_1/f_2 point and normalized by $f_1 + f_2 = f_m$.

If f_1 increases to f_{1n} , then n increases. Simultaneously the band shifts to the left, as shown by the dotted line. Also, since the new f_m is larger, the percentage bandwidth decreases slightly. If f_1 decreases to f_{1n} , the reverse occurs, as shown by the dot-dash line. If f_1 is held fixed and f_2 is increased to f_{2n} , n decreases and the bandwidth shifts left and decreases (dashed line).

Locus approximates a hexagon

Again, a decrease in f_2 causes the reverse to occur. Now for each value of f_2 , you can also vary f_1 ; this results in a region bordered by a hexagon. Because of the normalization by the



f_1	400±10	220±10	150±10	100±10
f_2	1127±30	620±30	423±30	281±30
f_0	1527±40	840±40	573±40	381±40

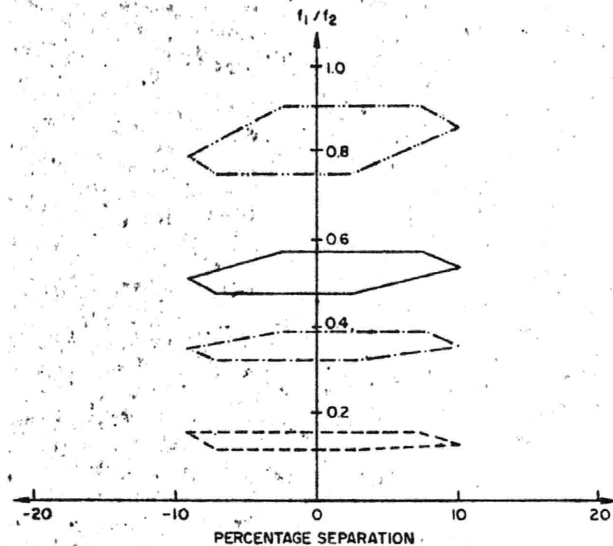
13. With f_1/f_2 constant, the spur map shows that a smaller percentage bw (or higher f_0) is better.

local value of f_0 , the sides of the hexagon are not straight lines but hyperbolic arcs. However, the curvature is very slight, and for all practical purposes, the sides can be thought of as straight lines.

Another program for the HP 9810 calculator plots the hexagon on the spur chart (Fig. 9). Fig. 10 shows an example generated by this program. To help illustrate the earlier discussion, the mapping of the output bandwidth for each set of f_1 and f_2 values is also superimposed ($f_1 = 16, 17, 18, \dots, 24$; $f_2 = 30, 39, 48, 57$). If you change f_1 and f_2 in infinitesimal steps, rather than the 1 and 9-MHz steps of the example, you end up with a region whose perimeter is defined by the hexagon shown.

An example outlines the use of this program as a design tool. For a proposed output bandwidth spanning 800 to 880 MHz, choose input frequencies between, say, 220 and 620 MHz to give a favorable f_1/f_2 value on the summing-mixer spur chart. Fig. 11 is a plot of the areas (through which spur lines cannot be tolerated) for five different bandwidth assignments for f_1 and f_2 . This graph clearly shows several important features.

As mentioned previously, the dimensions of the hexagon change with the ratio of input bandwidths. In the two cases of zero bandwidth, the hexagon degenerates to a quadrangle. The total horizontal excursion of the figures remains constant for a constant percentage output band-



f_1	100±10	220±10	290±10	380±10
f_2	740±30	620±30	550±30	460±30
f_0	840±40	840±40	840±40	840±40

14. This map for a constant output bandwidth suggests that f_1/f_2 should be kept low.

width. Thus, for a given acceptable harmonic spur level and absolute output bandwidth, a minimum center frequency for the output signal is defined for either sum or difference mixing.

Furthermore areas free of spurs for various ranges of f_1/f_2 indicate suitable ratios of the input signals. Minimum f_1/f_2 ranges are obtained when the lower input signal is given the smaller bandwidth. (This is also a favorable condition for realizable filters in a network of mixers.) In some applications the shape of the hexagon may be important in the elimination of spurs. Again, this can be adjusted with the ratio of the absolute bandwidths of the input signals.

Exact relations between frequency assignment parameters have not been established. But Figs. 12 through 14 provide examples of the trends. These confirm what intuition suggests. The smallest region is of general interest when you map the output bandwidth on the spur chart.

Fig. 11 shows that it is preferable to have wider bandwidths with the higher mixing frequency than vice versa. Figs. 12 and 13 show that a smaller percentage output bandwidth is superior. Fig. 14 suggests that you should choose low f_1/f_2 ratios, but this suggestion is tempered by the actual bandwidths and the spurs that appear, as shown in Figs. 1 to 4. Graphs for difference mixers exhibit similar trends. ■■

References

1. Olson, W. R. and Salcedo, R. V., "Mixer Frequency Charts," *Frequency*, March-April, 1966.