PHASE NOISE MEASUREMENTS

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ABSTRACT

THIS PRESENTATION COVERS BASIC THEORY AND MEASUREMENT OF PHASE NOISE CHARACTERISTICS OF OSCILLATORS. THREE SEMI-AUTOMATED MEASUREMENT SYSTEMS WILL BE PRESENTED WITH EXAMPLES OF PHASE NOISE PLOTS. THE MEASUREMENT PROGRAMS WERE DEVELOPED BY W. D. SEAL WITH THE ASSISTANCE OF F. G. MENDOZA AND N. W. HUDSON.

SECOND DRAFT--------JANUARY, 1977

12 FREQUENCY STABILITY AND PHASE NOISE MEASUREMENTS

INTRODUCTION

The term frequency stability encompasses the concepts of random noise, intended and incidental modulation, and any other fluctuations of the output frequency of a device.

Frequency stability is the degree to which an oscillating source produces the same value of frequency throughout a specified period of time.

It is implicit in this general definition of frequency stability that the stability of a given frequency decreases if anything except a perfect sine function is the signal waveshape.

Long-term stability is usually expressed in terms of parts per million per hour, day, week, month or year. This stability represents phenomenon due to the aging process of circuit elements and of the material used in the frequency determining element.

Short-term stability relates to frequency changes of less than a few seconds duration about the nominal frequency.

The measurements of frequency stability can be accomplished in both the time domain and frequency domain.

In the frequency domain, measurements are performed using a spectrum analyzer which provides a frequency window following the detector.

In the time domain, measurements are performed with a gated counter which provides a time window following the detector.

Both frequency domain and time domain measurements are preferred in order to have a comprehensive and sufficient measure of frequency stability.

Phase noise is the term most widely used to describe the characteristic randomness of frequency stability and spectral purity usually refers to signal to phase noise ratio.

In this presentation we will attempt to conform to the definitions, symbols and terminology set forth in NBS Technical Notes 394 and 632 [4, 15].

12.1 BASIC CONCEPTS

A sine wave signal generator is one which has a voltage that changes in time exactly as "h" (amplitude) changes with angle ϕ. The signal is an oscillating signal because it repeats itself at the end of each period. The phase is the angle within a cycle corresponding to a particular "h" (amplitude).

In this presentation the Greek letter nu (ν) represents frequency for carrier-related measures. Modulation-related frequencies are designated (f).

The ideal (perfect) sine wave and the carrier related parameters are illustrated in Figure 12.1.

ν_0 - Average frequency (nominal frequency) of the signal

$\nu(t)$ - Instantaneous frequency of a signal = $\frac{1}{2\pi} \frac{d\phi}{dt}(t)$

V_0 - Nominal peak amplitude of a signal source output

τ_0 - Period of an oscillation

$$\tau_0 = \frac{1}{\nu_0} \tag{12.1}$$

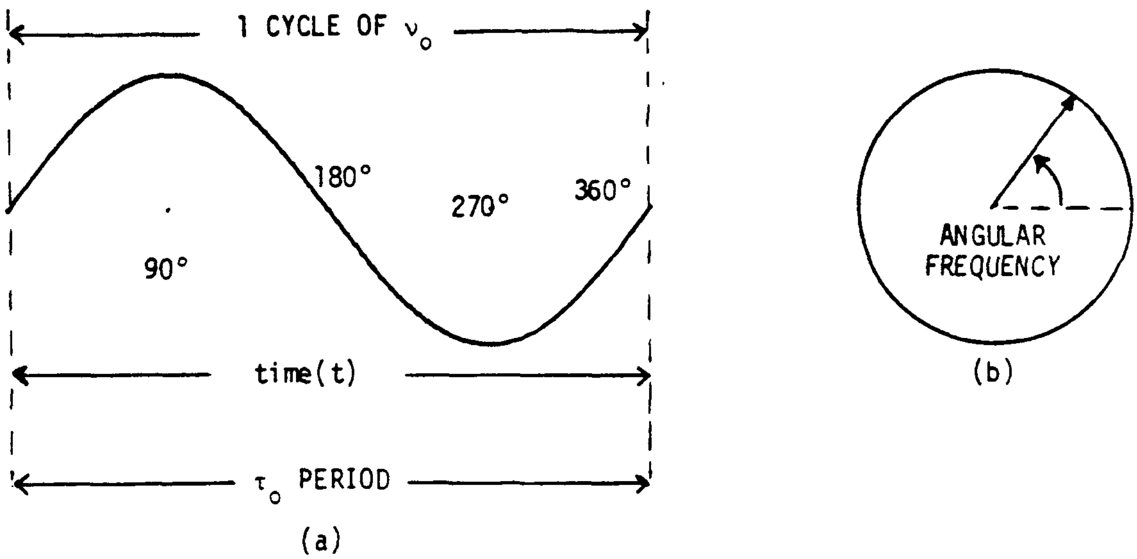

Figure 12.1 Illustration of Carrier-related Parameters of the Ideal Perfect Sine Function.

Ω - Signal (carrier) angular frequency (rate of change of phase with time)

$$\Omega = 2\pi\nu_0 \tag{12.2}$$

Ωt - Instantaneous angular frequency

ϕ - Phase in radians. The ideal phase is defined as,

$$\phi = 2\pi\nu_0 t \text{ (in radians)} \tag{12.3}$$

V(t) - Instantaneous output voltage of a signal. For the ideal sine wave signal of Figure 12.1.

$$V(t) = V_0 \text{ Sin } (2\pi\nu_0 t) \tag{12.4}$$

The basic relationship between phase (ϕ), frequency (ν_0), and time interval τ of the ideal (perfect) sine wave is

$$\phi = 2\pi\nu_0\tau \text{ (radians)} \tag{12.5}$$

$\Phi(t)$ - Instantaneous phase of the signal voltage $V(t)$. Defined for the ideal sine wave as

$$\Phi(t) = 2\pi\nu_0 t \tag{12.6}$$

i.e. $\Phi(t) = \phi$ for the ideal (perfect) signal.

Oscillators demonstrate noise which appears to be a superposition of causally generated signals and random, non-deterministic noises. The random noises include thermal noise, shot noise, noises of undetermined origin (such as flicker noise), and integrals of those noises. The end result is time dependent phase and amplitude fluctuations.

Figure 12.2 shows a simplified illustration of noise effects on a sine wave signal.

The instantaneous phase $\Phi(t)$ of $V(t)$ for the noisy signal is

$$\Phi(t) = 2\pi\nu_0 t + \phi(t) \tag{12.7}$$

Where $\phi(t)$ is the instantaneous phase fluctuation about the ideal phase $2\pi\nu_0\tau$ of Eq. 12.5.

In Figure 12.2a the data indicate the possible instantaneous noise voltages v_n. The dotted lines in Figure 12.2b indicate the boundaries of arbitrary fluctuations of the signal which is indicated by the solid line.

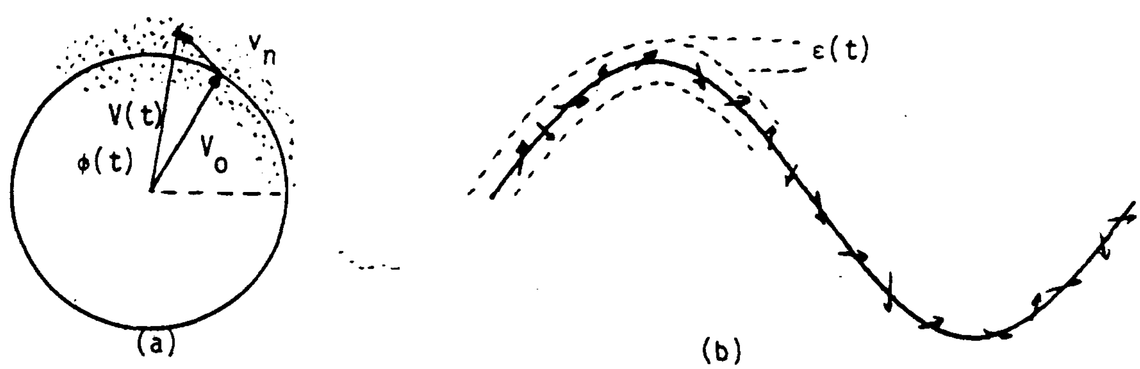

Figure 12.2 Magnified and Simplified Illustration of Boundaries of Instantaneous Phase, Amplitude, Frequency and Time Interval.

The instantaneous output voltage V(t) of a signal generator or oscillator is now

$$V(t) = [V_0 + \varepsilon(t)] \text{ Sin } [2\pi\nu_0 t + \phi(t)] \qquad (12.8)$$

Where V_0 and ν_0 are the nominal amplitude and frequency respectively and $\varepsilon(t)$ and $\phi(t)$ are the instantaneous amplitude and phase fluctuations of the signal.

It is assumed in Equation 12.8 that

$$\frac{\varepsilon(t)}{V_0} \ll 1 \text{ and } \frac{\dot{\phi}(t)}{\nu_0} \ll 1; \text{ for all } (t) \text{ and } \dot{\phi}(t) = \frac{d\phi}{dt} \qquad (12.9)$$

Equation 12.8 can also be expressed as

$$V(t) = [V_0 + \delta\varepsilon(t)] \text{ Sin } [2\pi\nu_0 t + \phi_0 + \delta\phi(t)] \qquad (12.10)$$

Where ϕ_0 is a constant, δ is the fluctuations operator, $\delta\varepsilon(t)$ and $\delta\phi(t)$ represent the fluctuations of signal amplitude and phase respectively.

Frequency fluctuations ($\delta\nu$) are related to phase fluctuations ($\delta\phi$) by

$$\delta\nu \equiv \frac{\delta\Omega}{2\pi} = \frac{1}{2\pi}\frac{d(\delta\phi)}{dt} \qquad (12.11)$$

i.e., radian frequency deviation is equal to the rate of change of phase deviation (the first time derivative of the instantaneous phase deviation).

The fluctuations of time interval ($\delta\tau$) are related to fluctuations of phase $\delta\phi$ by

$$\delta\phi = (2\pi\nu_0)\delta\tau \qquad (12.12)$$

y is defined as the fractional frequency fluctuation or fractional frequency **deviation.** It is the value of $\delta\nu$ normalized to the average (nominal) signal **frequency** ν_0.

$$y = \frac{\delta\nu}{\nu_0} \quad \text{(dimensionless)} \qquad (12.13)$$

y(t) is the instantaneous fractional frequency deviation from the nominal **frequency** ν_0.

12.2 SPECTRAL DENSITY

The frequency of a noisy oscillator is a combination of many different frequencies. Therefore, the addition of all Fourier frequencies produces the original signal. The modulation-related measures are f, the Fourier frequency variable and $\omega = 2\pi f$, the angular Fourfier frequency variable.

Suppose we consider that any signal that is produced which is not at the nominal frequency ν_0 is noise. The simplified illustration of Figure 12.3 shows a sine wave signal which is perturbed for a short instant by noise. In the first quadrant the waveform produces a change in amplitude (Δh) with a change in time interval (Δt). This corresponds to the nominal frequency of the sine wave. In the perturbed area in the second quadrant the Δh and Δt relationships correspond to other frequencies as represented by ν_1 and ν_2 in Figure 12.3.

The frequencies ν_1 and ν_2 are being produced for a given instant of time. This amount of time the signal spends in producing another frequency is referred to as the probability density at ν, (ν_1 and ν_2) relative to ν_0. The frequency domain plot is illustrated in Figure 12.3b. A graph of these probability densities over a period of time produces a continuous line. The resulting graph is called the POWER SPECTRAL DENSITY. The units of power spectral density are power per hertz. Since the units are power per hertz, a plot of power spectral density obtained from amplitude (voltage) measurements requires that the voltage measurements be squared.

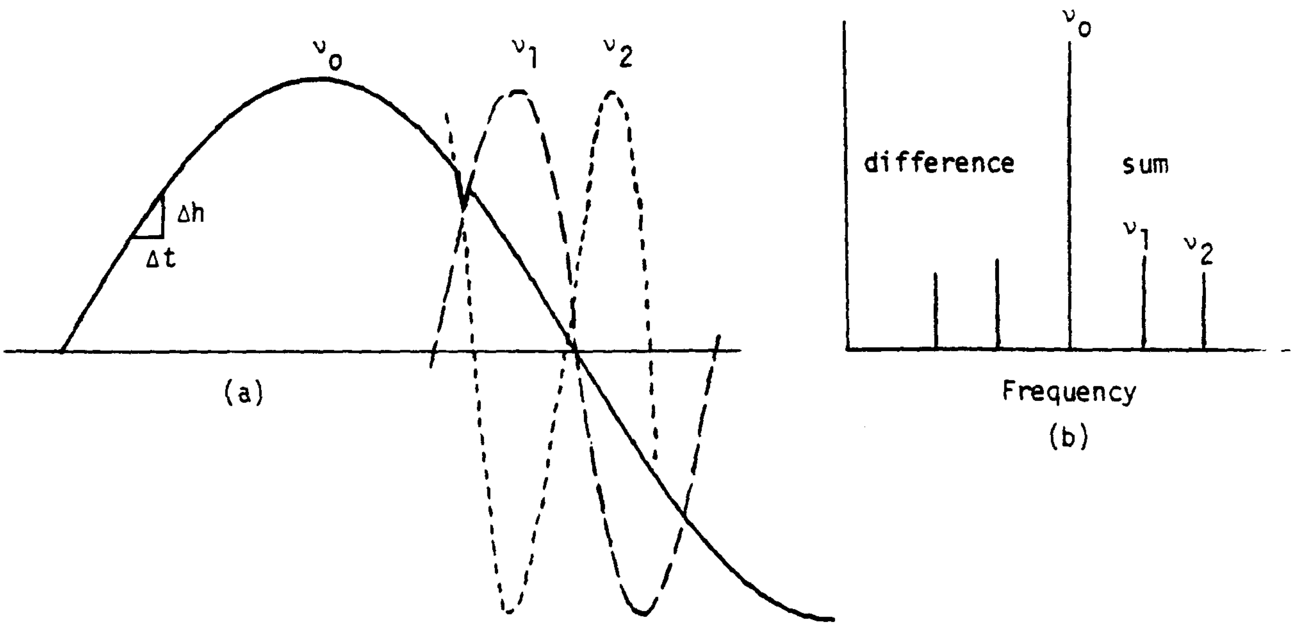

Figure 12.3 Simplified Illustration of Probability Density (a) Sine Wave with Noise Generated Frequencies ν_1 and ν_2. (b) Probability Density Plot of ν_1 and ν_2 relative to ν_o.

The <u>spectral density</u> is the distribution of the energy versus frequency as illustrated in Figure 12.4. This is a two-sided spectral density plot since the range of Fourier frequencies is from minus infinity to plus infinity.

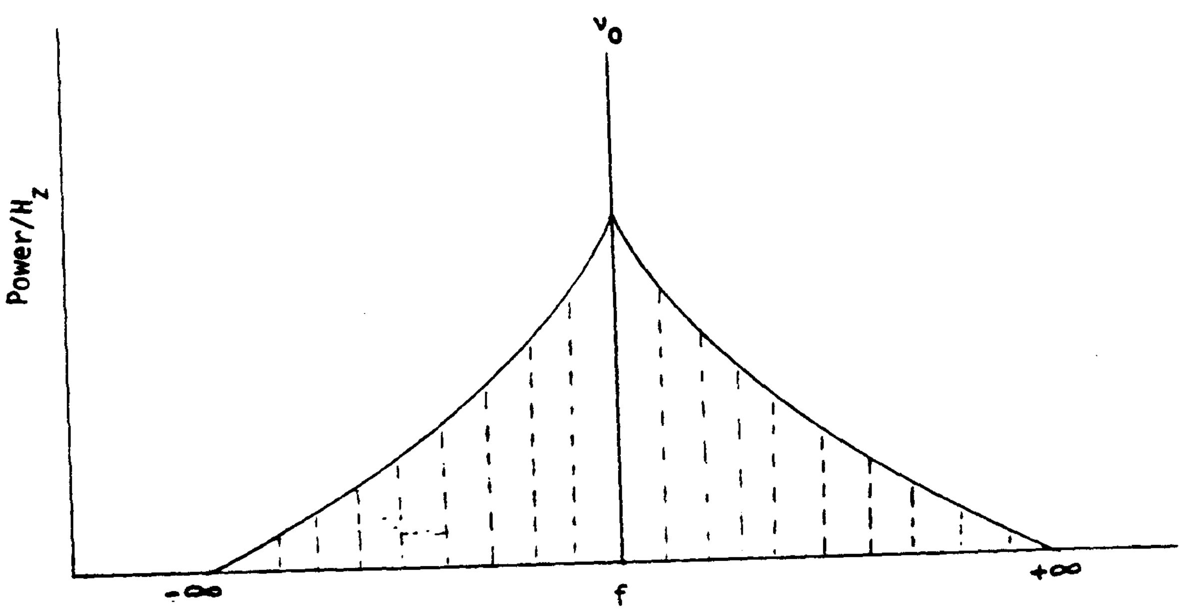

Figure 12.4. Illustration of Power Density Plot.

The notation $S_g(f)$ *represents the spectral density of fluctuations of any specified time dependent quantity (g(t)).*

$S_g(f)$ - The one-sided spectral density on a per hertz of bandwidth density basis. This means that the total mean-square fluctuation (the total variance) of the (pure real) function is given mathematically by the integral of the spectral density over the total defined range of Fourier frequency *f*.

$$\int_{0}^{+\infty} S_g(f)\, df \qquad (12.14)$$

12.3 SPECTRAL DENSITIES IN THE FREQUENCY DOMAIN

The spectral density $S_y(f)$ of the instantaneous fractional frequency fluctuations y(t) is defined as a measure of frequency stability [3]. $S_y(f)$ is the one-sided power spectral density of frequency fluctuations on a "per hertz" basis. i.e., the dimensionality is Hz^{-1}.

$$S_y(f) = \frac{S_{\delta\nu}(f)}{\nu_o^2} \quad Hz^{-1} \qquad (12.15)$$

$S_{\delta\nu}(f)$, in Hz^2/Hz, is the spectral density of frequency fluctuations $\delta\nu$. *It is calculated as* $(\delta\nu_{rms})^2$*/Bandwidth used in the measurement of* $\delta\nu_{rms}$.

The power spectral density of phase fluctuations is a normalized frequency domain measure of phase fluctuation sidebands defined as follows:

$S_{\delta\phi}(f)$, in radians squared per hertz, is the one-sided power spectral density of the phase fluctuations on a "per hertz" basis.

$$S_{\delta\phi}(f) \equiv \frac{\text{Power density of phase fluctuations at f in one hertz}}{\text{Total signal power}} \qquad (12.16)$$

The phase and frequency fluctuation power spectral densities are related by:

$$S_{\delta\phi}(f) = \left[\frac{\nu_o^2}{f^2}\right] S_y(f) \quad \text{radians}^2/\text{Hz} \qquad (12.17)$$

$S_{\delta\Omega}(f)$, in $(rad/s)^2/Hz$, is the spectral density of angular frequency fluctuations $\delta\Omega$. The defined spectral densities have the following interconnecting relationships.

$$S_{\delta\nu}(f) = \nu_0^2 \, S_y(f) = (1/2\pi)^2 \, S_{\delta\Omega}(f) = f^2 \, S_{\delta\phi}(f) \qquad Hz^2/Hz \quad (12.18)$$

or

$$S_y(f) = \frac{S_{\delta\nu}(f)}{\nu_0^2} = \left(\frac{1}{2\pi}\right)^2\left(\frac{1}{\nu_0^2}\right) S_{\delta\Omega}(f) = \frac{f^2}{\nu_0^2} S_{\delta\phi}(f) \qquad Hz^{-1} \quad (12.19)$$

The National Bureau of Standards recommends a definition of frequency stability which relates the actual sideband power of phase fluctuations with respect to the carrier power level. The defined definition is called Script $\mathcal{L}(f)$

Script $\mathcal{L}(f)$ is defined as the ratio of the power in one sideband, referred to the input carrier frequency, on a per hertz of bandwidth spectral density basis, to the total signal power, at Fourier freouency f from the carrier, per one device. It is a normalized frequency domain measure of phase fluctuation sidebands expressed as dB relative to the carrier per Hz.

$$\mathcal{L}(f) = \frac{\text{Power Density (one phase modulation sideband)}}{\text{Carrier Power}} \quad dBc/Hz \; (12.20)$$

For the condition that the phase fluctuations occurring at rates (f) and faster are small compared to one radian, a good approximation is,

$$\mathcal{L}(f) = \frac{S_{\delta\phi}(f)\ \text{(one unit)}}{2\ \text{radians}^2} \qquad Hz^{-1} \qquad (12.21)$$

If the small angle condition is not met, Bessel function algebra must be used to relate $\mathcal{L}(f)$ to $S_{\delta\phi}(f)$.

The NBS defined spectral density is usually expressed in decibels relative to the carrier per hertz (dBc/Hz) and is calculated as,

$$\mathcal{L}(f) = 10 \log \left[\frac{S_{\delta\phi}(f)\ \text{(one unit)}}{2\ \text{rad}^2}\right] dBc/Hz \qquad (12.22)$$

<u>It is very important to note that the theory, definitions and equations previously set forth relate to a SINGLE DEVICE</u>.

$S_{\delta\tau}(f)$, in seconds squared per hertz, is the spectral density of time interval fluctuations. $\delta\tau = \delta\phi/(2\pi\nu_0)$.

The spectral densities of frequency, phase and time interval have the following interconnecting relationships.

$$S_{\delta\phi}(f) = (2\pi\nu_0)^2 \, S_{\delta\tau}(f), \; rad^2/H_z \qquad (12.23)$$

$$S_{\delta\tau}(f) = \left(\frac{1}{2\pi f}\right)^2 S_y(f), \; s^2/H_z \qquad (12.24)$$

12.4 NOISE PROCESSES

The spectral density plot of a typical oscillator's output is usually a combination of different noise processes. It is very useful and meaningful to categorize these processes since the first job in evaluating a spectral density plot is to determine which type of noise exists for the particular range of Fourier frequencies.

The two basic categories of noise are the discrete frequency noise and the power-law noise process.

Discrete frequency noise - a type of noise in which there is a dominant observable probability i.e. deterministic in that they can usually be related to the mean frequency, power line frequency, vibration frequencies or AC magnetic fields, or Fourier components of the nominal frequency.

Discrete frequency noise is illustrated in the frequency domain plot of Figure 12.5

These frequencies can have their own spectral density plots which can be defined as noise on noise.

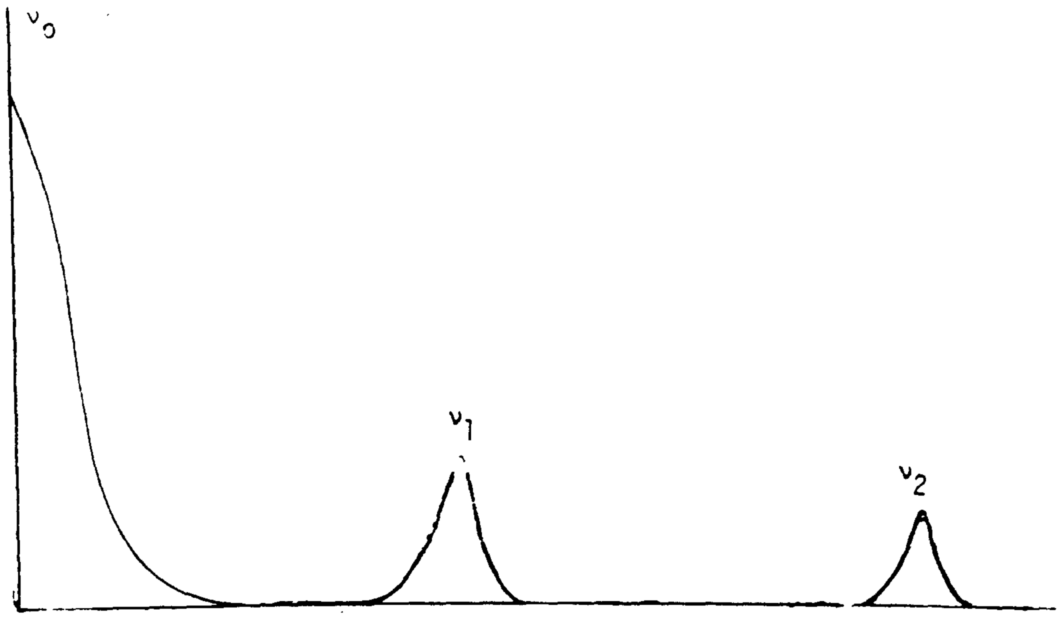

Figure 12.5 Basic Illustration of Discrete Frequency Signal Display.

Power-law noise process - Types of noise which produce a certain slope on the one sided spectral density plot. They are characterized by their dependence on frequency. The spectral density plot of a typical oscillator output is usually a combination of the different power-law processes.

In general we can classify the power-law noise processes into five categories. These five processes are illustrated in Figure 12.6 which can be referred to with respect to the following description of each process.

1. RANDOM WALK FM (random walk of frequency). The plot goes down as $1/f^4$.

   This noise is usually very close to the carrier and is difficult to measure. It is usually related to the oscillator's physical environment (mechanical shock, vibration, temperature, or other environmental effects).

2. FLICKER FM (flicker of frequency). The plot goes down as $1/f^3$.

This noise is typically related to the physical resonance mechanism of the active oscillator or the design or choice of parts used for the electronic or power supply, or even environmental properties. The time domain frequency stability over extended periods is constant. In high quality oscillators this noise may be marked by white FM ($1/f^2$) or flicker phase modulation ϕM ($1/f$). If may be masked by drift in low quality oscillators.

3. White FM (white of frequency), Random Walk of Phase. Plot goes down as $1/f^2$.

A common type of noise found in passive-resonator frequency standards. Cesium and rubidium frequency standards have white FM noise characteristics since the oscillator (usually quartz) is locked to the resonance feature of these devices. This noise gets better as a function of time until it (usually) becomes flicker FM ($1/f^3$) noise.

4. FLICKER ϕM (flicker(modulation) of phase). The plot goes down as 1/f.

This noise may relate to the physical resonance mechanism in an oscillator. It is common in the highest quality oscillators. This noise can be introduced by noisy electronics - amplifiers necessary to bring the signal amplitude up to a usable level, and frequency multipliers. This noise can be reduced by careful design and hand-selecting all components.

5. WHITE ϕM (white of phase). White phase noise plot is flat f^0.

Broadband phase noise is generally produced in the same way as flicker ϕM ($1/f$). Late stages of amplification are usually responsible. This noise can be kept low by careful selection of components and narrow band filtering at the output.

The power-law processes are illustrated in Figure 12.6.

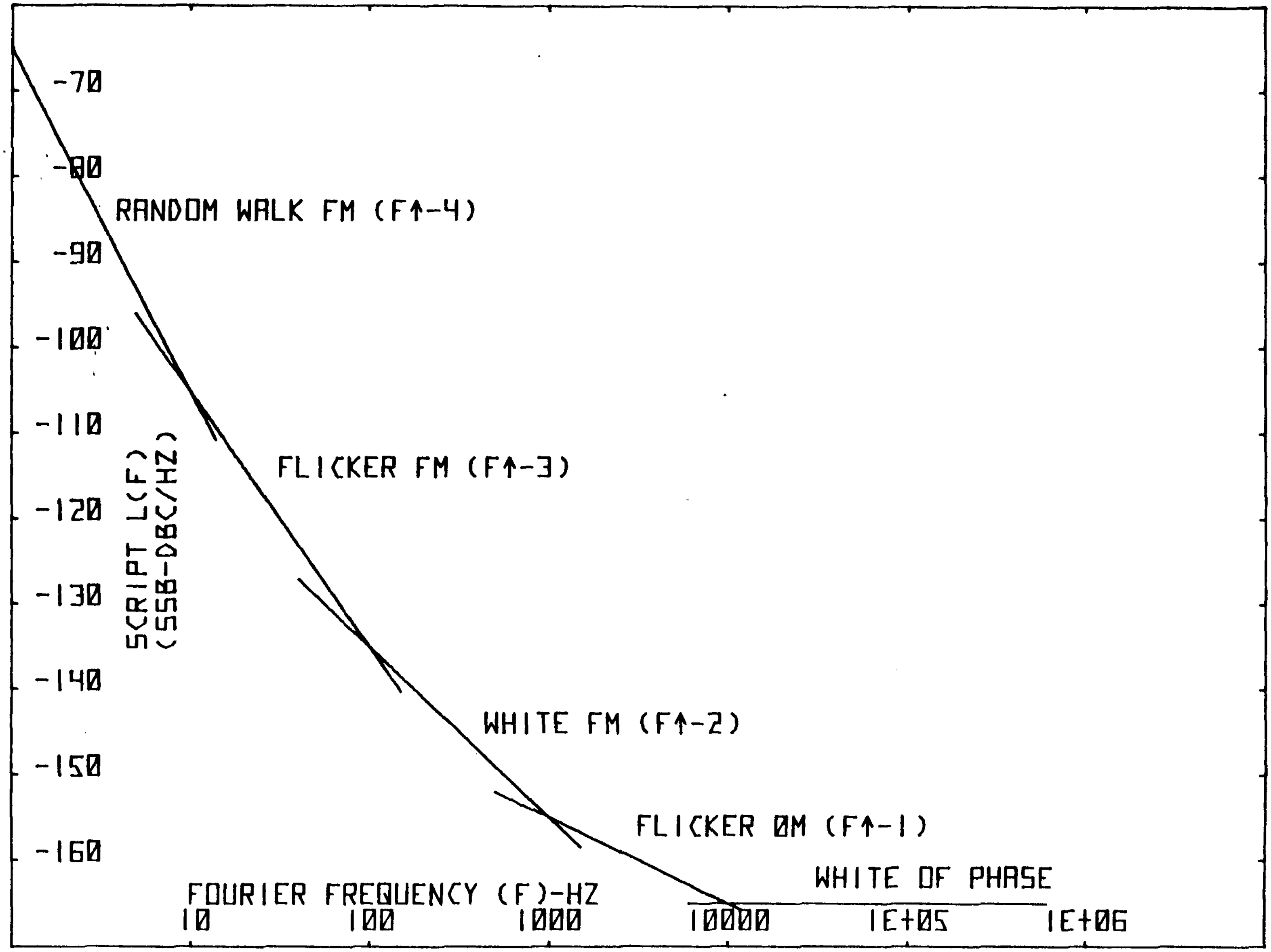

FIGURE 12.6 SPECTRAL DENSITY PLOT OF PHASE ILLUSTRATING THE FIVE POWER-LAW NOISE PROCESS.

12.5 BASIC MODULATION THEORY AND SPECTRAL DENSITY RELATIONSHIPS

Applying a sinusoidal frequency modulation (f_m) to a sinusoidal carrier frequency (ν_o) produces a wave that is sinusoidally advanced and retarded in phase as a function of time. The instantaneous voltage is expressed as,

$$V(t) = V_o \text{ Sin } [2\pi\nu_o t + \Delta\phi \text{ Sin } 2\pi f_m t] \qquad (12.25)$$

Where $\Delta\phi$ is the peak phase deviation caused by the modulation signal.

The first term inside the brackets represents the linearly progressing phase of the carrier. The second term is the phase variation (advancing and retarded) from the linearly progressing wave.

The effects of modulation can be expressed as residual fm noise or as single side-band phase noise. For modulation by a single sinusoidal signal the peak frequency deviation of the carrier (ν_o) is

$$\Delta\nu_o = \Delta\phi \cdot f_m \qquad (12.26)$$

$$\Delta\phi = \frac{\Delta\nu_o}{f_m} \qquad \text{(peak)} \qquad (12.27)$$

where f_m is the modulation frequency.

This ratio of peak frequency deviation to modulation frequency is called Modulation index (m) so that $\Delta\phi = m$ and

$$m = \frac{\Delta\nu_o}{f_m} \qquad (12.28)$$

The frequency spectrum of the modulated carrier contains frequency components (side-bands) other than the carrier. For small values of modulation index ($m<<1$), as is the case with random phase noise, only the carrier and first upper and lower sidebands are significantly high in energy.

The ratio of the amplitude of either side-band (single side-band) to the amplitude of the carrier is

$$\frac{V_{SB}}{V_o} = \frac{m}{2} \tag{12.29}$$

This ratio is expressed in dB and referred to as dBc (dB below the carrier for the given bandwidth).

$$dBc = \left.\frac{V_{SB}}{V_o}\right|_{dB} = 20 \log \frac{m}{2} = 20 \log \frac{\Delta\nu_o}{2f_m} \tag{12.30}$$

which can also be expressed as

$$dBc = 10 \log \left(\frac{m}{2}\right)^2 = 10 \log \left(\frac{\Delta\nu_o}{2f_m}\right)^2 \tag{12.31}$$

The NBS defined spectral density of phase fluctuations is $\mathcal{L}(f)$ in dBc/Hz. Therefore,

$$\left(\frac{m}{2}\right)^2 = 10^{\mathcal{L}(f)/10} \tag{12.32}$$

The modulation index (m) can now be expressed in terms of the spectral density of phase fluctuations,

$$m = \sqrt{2 \times 10^{\mathcal{L}(f)}} = \sqrt{S_{\delta\phi}(f)} \quad \text{radians} \tag{12.33}$$

As an example, assume that $\mathcal{L}(f)$ is -83 dBc/Hz,

$$m = \sqrt{2 \times 10^{-83/10}} = 10^{-4} \quad \text{radians} \tag{12.34}$$

And

$$S_{\delta\phi}(f) = m^2 = (10^{-4} \text{ radians})^2 \tag{12.35}$$

which is equal to -80 dB relative to one square radian.

If the frequency deviation is given in terms of its rms value then,

$$\Delta\nu_{rms} = \frac{\Delta\nu_o}{\sqrt{2}} \tag{12.36}$$

Equation 12.31 now becomes

$$\left.\frac{V_{SB}}{V_o}\right|_{dB} = \mathcal{L}(f) = 20 \log \frac{\Delta\nu_{rms}}{\sqrt{2} f_m} = 10 \log \left(\frac{\Delta\nu_{rms}}{\sqrt{2} f_m}\right)^2 \tag{12.37}$$

The ratio of single sideband to carrier power in dBc/Hz is

$$\mathcal{L}(f) = 20 \log \left[\frac{\Delta\nu_{rms}}{f_m}\right] - 3 \text{ dB} \tag{12.38}$$

and

$$S_{\delta\phi}(f) = 20 \log \left[\frac{\Delta\nu_{rms}}{f_m}\right] \tag{12.39}$$

The relationships of modulation index, rms frequency deviation, peak frequency deviation, and spectral density of phase fluctuations are shown in the following equation.

$$\frac{m}{2} = \frac{\Delta\nu_{rms}}{\sqrt{2}\, f_m} = \sqrt{10^{\mathcal{L}(f)/10}} = \frac{1}{2}\sqrt{S_{\delta\phi}(f)} \qquad (12.40)$$

Relative scale plots of $\mathcal{L}(f)$ in dBc/Hz, spectral density of phase in radians squared per hertz, spectral density of phase in dB realtive to one square radian, and modulation index in radians are shown in Figure 12.7.

Previous discussions indicated that when noise modulation indices are small, correlation noise can be neglected. (Two signals are uncorrelated if their phase and amplitudes have different time distributions so that they do not cancel in a phase detector). It is useful to separate noise side-bands into amplitude modulation (AM) and frequency modulation (FM) components and to discuss noise as a modulation phenomenon.

Noise side-bands can be thought of as arising from a composite of low frequency signals. Each of these signals modulate the carrier producing components in both side-bands separated from the carrier by the modulation frequency.

Figure 12.8 illustrates that a signal, such as a single upper side-band, can be represented by a pair of symmetrical side-bands (pure AM), and a pair of antisymmetrical side-bands (pure FM). The resultant FM noise vector lies at right angles to the carrier.

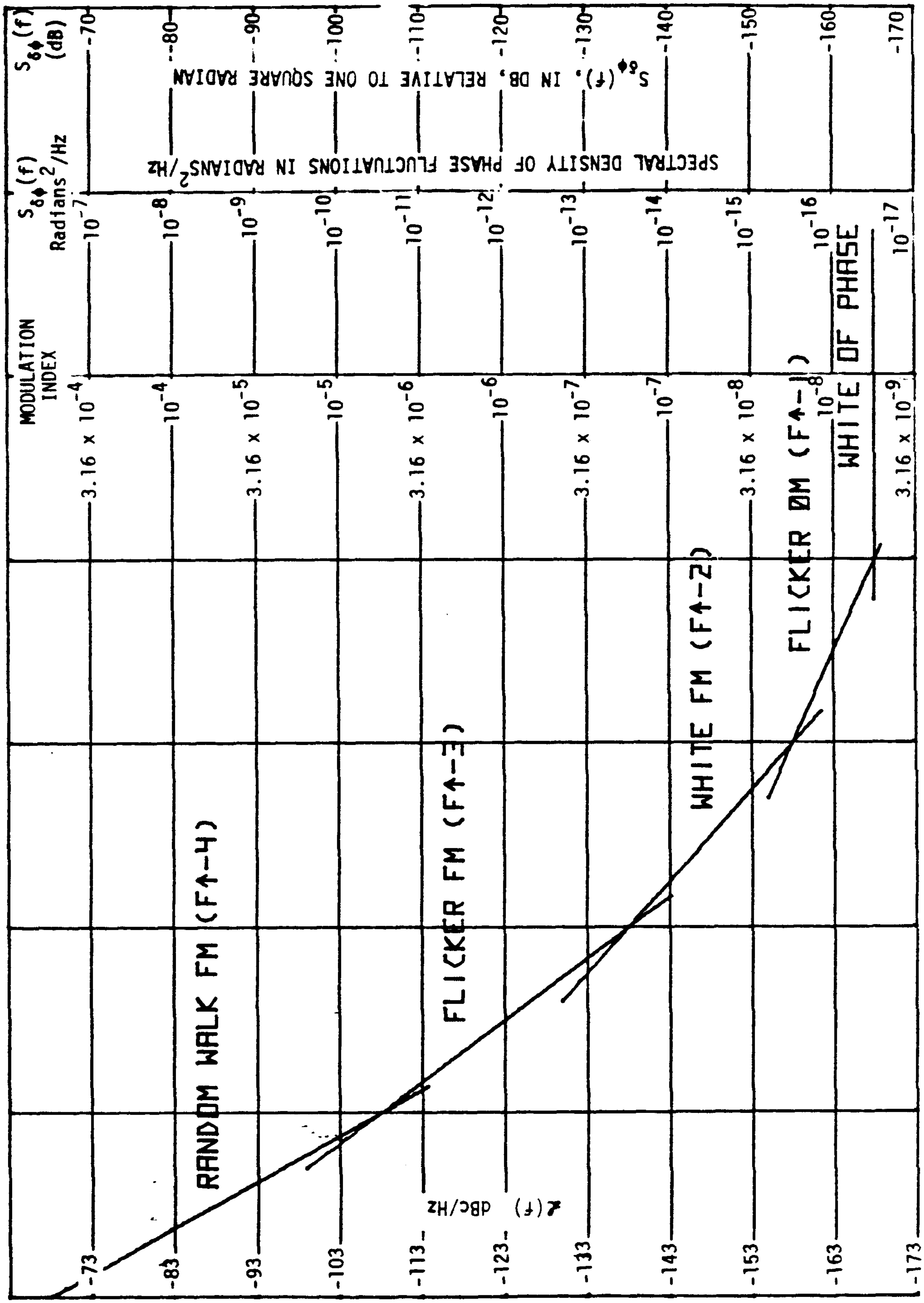

FIGURE 12.7 RELATIVE SCALE PLOTS OF SPECTRAL DENSITIES

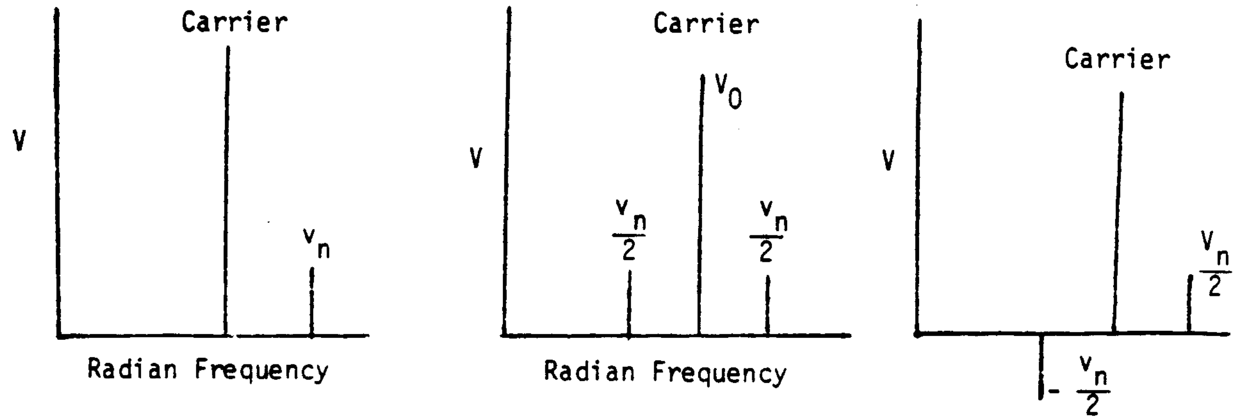

FIGURE 12.8. (a) Carrier and single upper Side-band signal. (b) Symmetrical Side-bands (pure AM). (e) Antisymmetrical pair of Side-bands (pure FM).

Amplitude fluctuations can be measured with a simple amplitude detector such as a crystal. Phase or frequency fluctuations can be detected with a discriminator. Frequency modulation (FM) noise or rms frequency deviation can also be measured with an amplitude modulation (AM) detection system after the FM or rms frequency deviations are converted to amplitude modulation.

Figure 12.9a illustrates the relationships referred to in Figure 12.8. Under this condition the AM is detected with an amplitude detector as indicated.

Figure 12.9b shows that FM to AM conversion as well as AM to FM conversion can be obtained if the carrier can be adjusted to quadrature phase with the FM as shown by the 90° advance in phase of the carrier. FM to AM conversion is shown on the left and AM to FM conversion is on the right. The FM deviation is now measurable as an AM variation on the 90° advanced (or retarded) carrier signal.

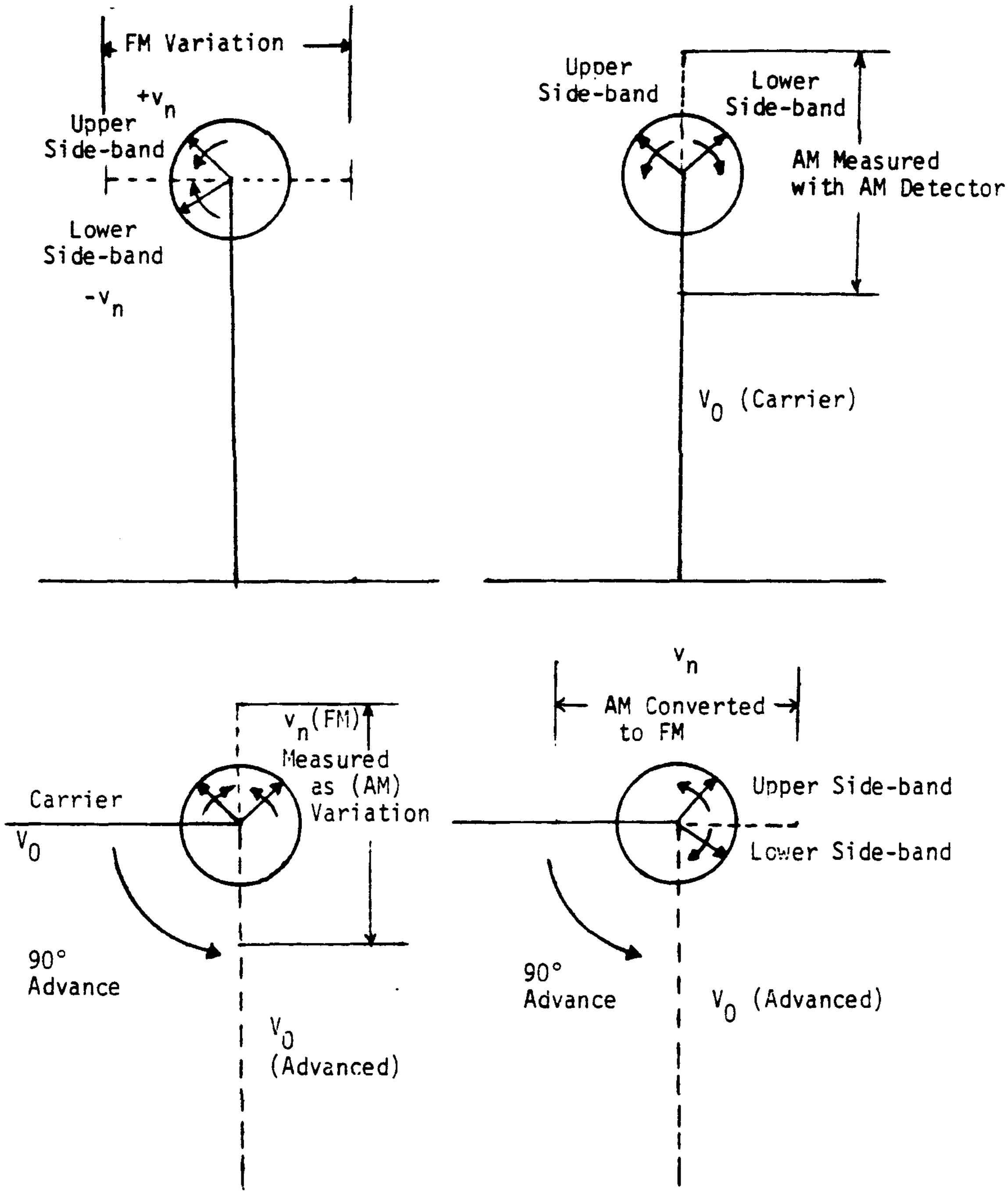

FIGURE 12.9. (a) Relationships of the Carrier to the AM and FM noise Side-bands.

(b) Carrier advanced 90° to obtain FM to AM Conversion on the left and AM to FM Converison on the right.

12.6 TWO-OSCILLATOR TECHNIQUE (Phase Noise Measurements in the Frequency Domain)

A functional block diagram of the measurement system is shown in Figure 12.10. NBS has performed phase noise measurements using this basic type system since 1967. The signal level and sideband levels can be measured in terms of voltage or power.

Assume that the reference oscillator is perfect (no phase noise) and that it can be adjusted in frequency. Also, assume that both oscillators are extremely stable so that phase quadrature can be maintained without the use of an external phase-lock loop.

The double balanced mixer acts as a phase detector so that when two signals are identical in frequency and are in phase quadrature, the output is approximately zero volts dc. The mixer output is a small fluctuating voltage δv centered on zero volts. This small fluctuating voltage represents the phase modulation PM sideband component of the signal because, due to the quadrature of the signals at the mixer input, the mixer converts the amplitude modulation AM sideband components to FM and at the same time it converts the phase modulation PM sideband components to AM and these AM components are detected with an amplitude detector [9].

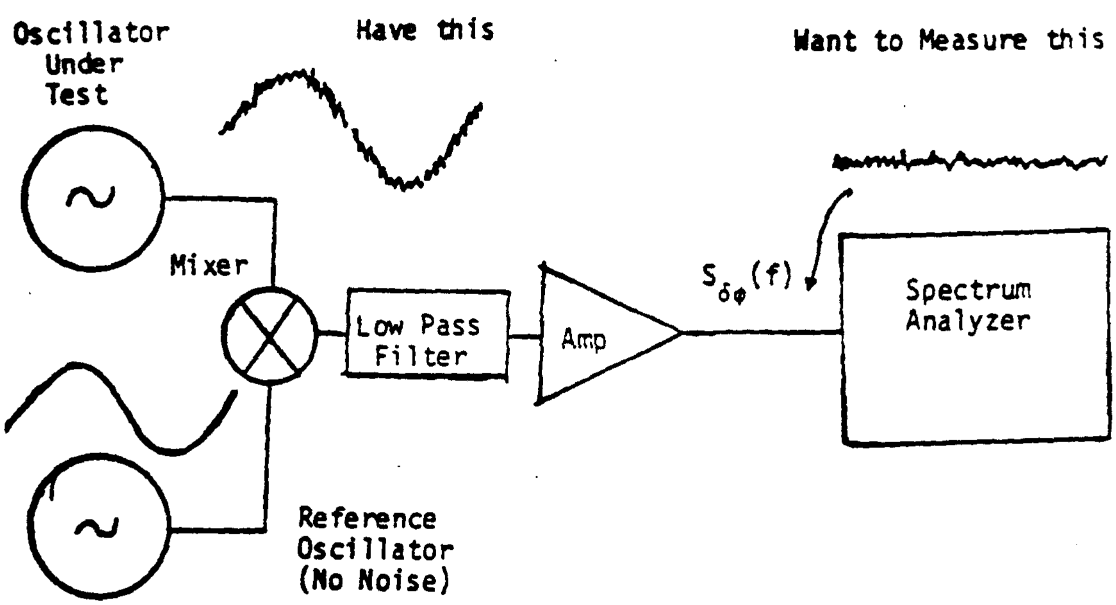

FIGURE 12.10. FUNCTIONAL BLOCK DIAGRAM OF A PHASE NOISE MEASUREMENT SYSTEM USING THE TWO-OSCILLATOR TECHNIQUE.

If the two oscillator signals applied to the double balanced mixer of Figure 12.10 are slightly out of zero beat, a slow sinusoidal voltage with a peak-to-peak voltage V_{ptp} can be measured at the mixer output. If these same signals are returned to zero beat and adjusted for phase quadrature, the output of the mixer is a small fluctuating voltage (δv) centered at zero volts. If the fluctuating voltage is small compared to $V_{ptp}/2$, the phase quadrature condition is being closely maintained and the "small angle" condition is being met. Phase fluctuations between the test and reference signals (phases) are,

$$\delta\phi = \delta(\phi_t - \phi_r) \qquad (12.41)$$

These phase fluctuations produce voltage fluctuations at the output of the mixer,

$$\delta v = \frac{V_{ptp}}{2}\,\delta\phi \qquad (12.42)$$

Where phase angles are in radian measure and Sin $\delta\phi = \delta\phi$ for small $\delta\phi$ ($\delta\phi \ll 1$ radian).

Solving for $\delta\phi$, squaring both sides and taking a time average gives,

$$<(\delta\phi)^2> = 4\,\frac{<(\delta v)^2>}{(V_{ptp})^2} \qquad (12.43)$$

Where the brackets represent the time average.

For the sinusoidal beat signal,

$$(V_{ptp})^2 = 8(V_{rms})^2 \qquad (12.44)$$

The mean-square fluctuations of phase $\delta\phi$ and voltage δv interpreted in a spectral density fashion gives,

$$S_{\delta\phi}(f) = \frac{S_{\delta v}(f)}{2(V_{rms})^2} \qquad (12.45)$$

Here, $S_{\delta v}(f)$, in volts squared/Hz, is the spectral density of the voltage fluctuations at the mixer output. Since the spectrum analyzer measures rms voltage, the noise voltage is in units of "volts per $\sqrt{Hz}$ (volts per root hertz) which means volts per $\sqrt{Bandwidth}$. Therefore,

$$S_{\delta v}(f) = \left[\frac{\delta v_{rms}}{\sqrt{B}}\right]^2 = \frac{(\delta v_{rms})^2}{B} \quad volts^2/Hz \quad (12.46)$$

Where B is the noise power bandwidth used in the measurement.

Since it was assumed that the reference oscillator did not contribute any noise, the voltage fluctuations δv_{rms} represent the oscillator under test and the spectral density of the phase fluctuations in terms of the voltage measurements performed with the spectrum analyzer is,

$$S_{\delta\phi}(f) = \frac{1}{2}\left[\frac{(\delta v_{rms})^2}{B(V_{rms})^2}\right] \quad radians^2/Hz \quad (12.47)$$

Equation 12.45 is sometimes expressed as [3],

$$S_{\delta\phi}(f) = \frac{S_{\delta v}(f)}{K^2} \quad radians^2/Hz \quad (12.48)$$

Where K is the calibration factor in volts/radian. For sinusoidal beat signals, the peak voltage of the signal equals the slope of the zero crossing in volts/radian. Therefore, $(V_{peak})^2 = 2(V_{rms})^2$ which is the same as the denominator in Equation 12.45.

$S_{\delta\phi}(f)$ can be expressed in terms of decibels relative to one square radian by calculating 10 log $S_{\delta\phi}(f)$ of the previous equation.

$$S_{\delta\phi}(f) = [20 \log(\delta v_{rms}) - 20 \log(V_{rms}) - 10 \log(B) - 3\ dB]\ dBr \quad (12.49)$$

A correction of +2.5 dB is required for the Tracking Spectrum Analyzer used in these measurement systems.

The NBS defined spectral density of phase fluctuations differs by 3 dB and is therefore expressed as,

$$\mathcal{L}(f) = [\, 20 \log(\delta v_{rms}) - 20 \log(V_{rms}) - 10 \log(B) - 6 \text{ dB}] \text{ dBc/Hz} \quad (12.50)$$

Spectrum analyzer corrections must also be added to this equation.

SPECTRAL DENSITY OF AMPLITUDE FLUCTUATIONS [14]

The spectral density ($S_{\delta\varepsilon}(f)$) of the amplitude fluctuations of a signal follows the same general derivation previously given for the spectral density of phase fluctuations.

When two signals are slightly different in frequency, a slow, almost sinusoidal beat is produced at the mixer output and the peak-to-peak voltage swing is definded as (V_{ptp}).

If the two signals are now tuned to colinear phase (0 or 180° phase angle difference), the mixer output is a fluctuating voltage centered on $V_{ptp}/2$ volts. Remember that FM to AM conversion (to obtain measurements of $\delta\phi$) required that the two mixer input signals be in quadrature in which case the voltage fluctuation at the mixer output were centered at approximately zero volts dc.

In order to obtain linearity in the measurements of AM, and to make the measurement sensitive to the test oscillator only, the reference signal into the balanced mixer should be at least 10dB greater than the test oscillator signal input to the mixer.

Amplitude fluctuations $\delta\varepsilon$ produce voltage fluctuations at the mixer output.

$$\delta\varepsilon = \left(\frac{V_{ptp}}{2}\right) \frac{\delta\varepsilon}{V_o} \quad (12.51)$$

Squaring both sides and taking a time average,

$$<(\delta\varepsilon)^2> \;=\; 4V_o^{\,2}\left(\frac{<(\delta v)^2>}{(V_{ptp})^2}\right) \quad (12.52)$$

The spectral density is given by,

$$S_{\delta\varepsilon}(f) = \frac{V_o^2}{2} \left(\frac{S_{\delta V}(f)}{(v_{rms})^2}\right) \tag{12.53}$$

Since the sinusoidal beat signal,

$$(V_{ptp})^2 = 8(V_{rms})^2 \tag{12.54}$$

and where the spectral density of the voltage fluctuations δv is given by

$$S_{\delta v}(f) = \frac{(\delta v_{rms})^2}{B} \tag{12.55}$$

The NBS defined spectral density of amplitude fluctuations is,

$$\mathcal{M}(f) = \left(\frac{1}{2V_o^2}\right) S_{\delta\varepsilon}(f) \tag{12.56}$$

also

$$\mathcal{M}(f) = 2\left(\frac{v_{(\text{one unit})}}{V_{ptp}}\right)^2 = \frac{S_{\delta v}(f)}{4\ V_{rms}^2} \tag{12.57}$$

i.e. v is the measured value of δv_{rms}.

TWO NOISY OSCILLATORS

The measurement system of Fig. 12.10 yields the output noise from both oscillators. If the reference oscillator is superior in performance as assumed in the previous discussions, then one obtains a direct measure of the noise characteristics of the oscillator under test.

If the reference and test oscillators are the same type, a good approximation is to assume that the measured noise power is twice that which is associated with one oscillator. One is in error by no more than 3 dB even if one oscillator is the major source of noise. The equation for the spectral density of phase fluctuations is,

$$S_{\delta\phi}(f)\,(\text{measured}) = \frac{S_{\delta V}(f)\ \text{(two units)}}{2(V_{rms})^2} = 2.\,S_{\delta\phi}(f) \qquad (12.58)$$

The measured value is therefore divided by two in order to obtain the value for the single oscillator. This problem can be resolved when one has three oscillators that can be measured in all pair combinations.

THE PHASE-LOCK LOOP

A phase-lock loop is illustrated in Figure 12.11.

The phase-lock loop is an electronic servomechanism that acts to null out any phase error between the two inputs to the mixer (phase detector). The time constant of the loop can be adjusted as needed by varing amplifier gain and RC filtering within the loop.

A loose phase-lock loop is charaterized by:

1. The correction voltage varies as phase (in the short term) and phase variations are therefore observed directly.
2. The bandwidth of the servo response is small compared to the Fourier frequency to be measured.
3. The response time is very slow.

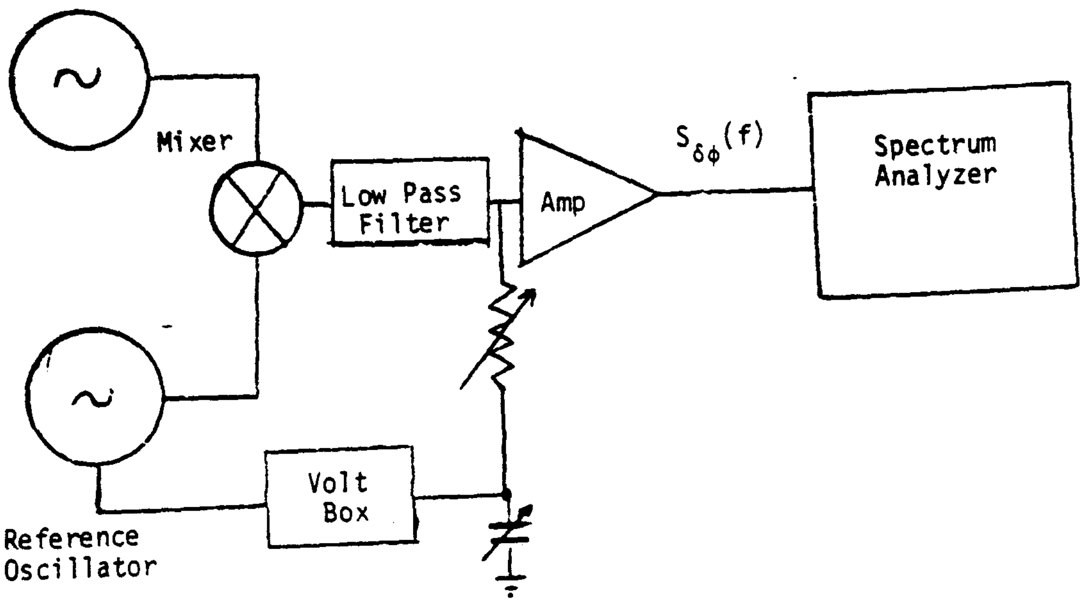

FIGURE 12.11. Phase Noise Measurement System with a Phase-Lock Loop.

The tight phase-lock loop is charaterized by:

1. The correction voltage of the servo loop varies as frequency.
2. The bandwidth of the servo response is relatively large.
3. The response time is much smaller than the smallest time interval (τ) at which measurements are performed.

Noise reduction is illustrated in Figure 12.12. Curve A shows the attenuation of the phase-lock loop. Curve B is a plot of the actual phase noise characteristic of the unit under test. Curve C shows the noise of reference oscillator (multiplied up to higher frequency) response including an increase in reference noise above 1MHz due to diode contribution in the multiplication process. The dotted line (D) illustrates the actual measurement plot which shows that the frequency components, of the unit under test, that fall within the loop bandwidth of the reference are reduced to the level of the noise of the reference. Also, outside the loop and above 1MHz, the reference noise is attenuated by the loop attenuation and does not contribute to the measurement.

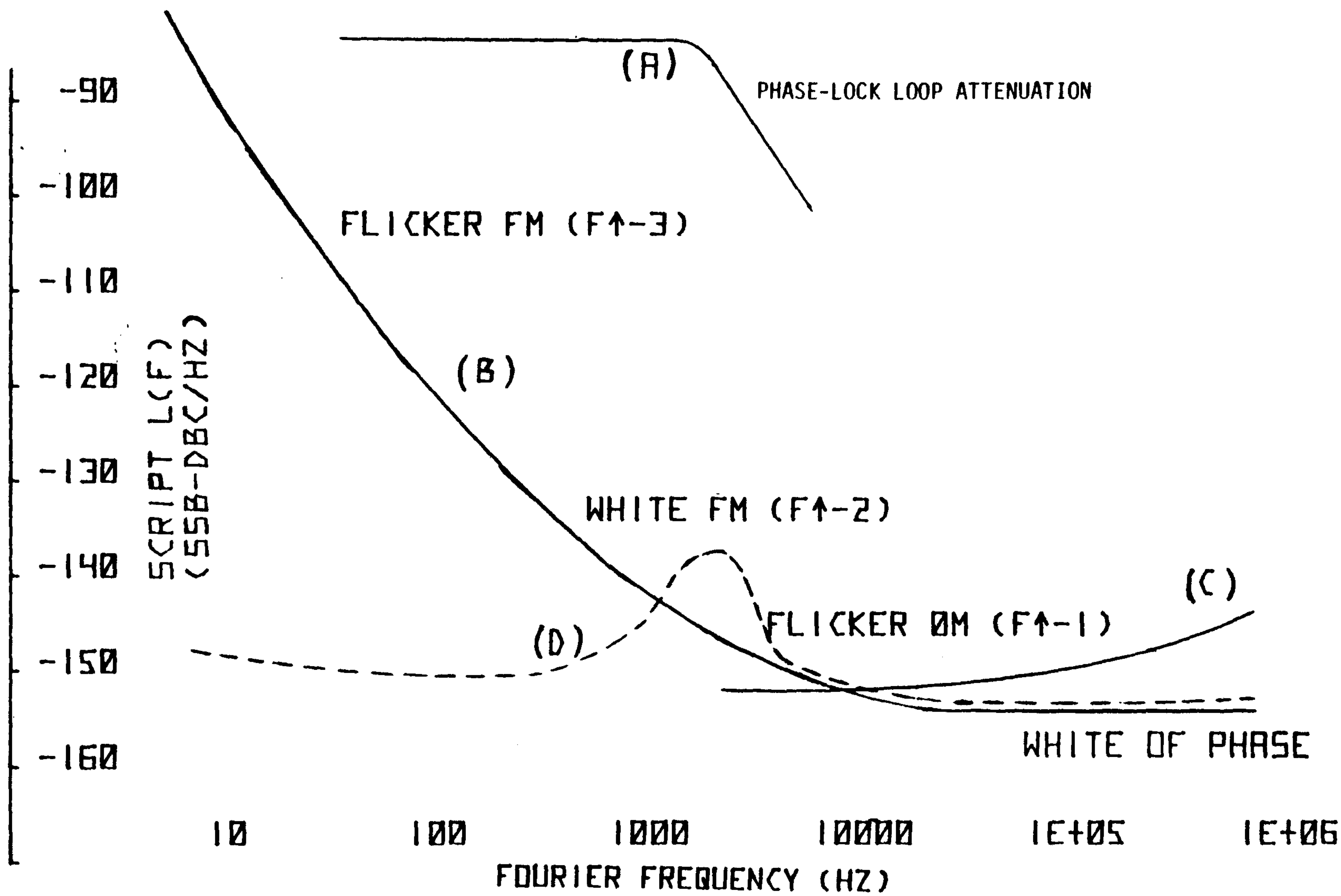

FIGURE 12.12 ILLUSTRATION OF PHASE-LOCK REDUCTION OF NOISE.

Typical plots of phase noise characteristics of devices are shown in Figure 12.13. The plotted calibration points were extracted from the continuous data plots obtained with the TRW Metrology measurement system.

Figure 12.13a is a plot obtained by measurements of two free-running 5MHz oscillators which had approximately equal noise characteristics. These units were very stable so that quadrature was maintained by manual adjustment of frequency. Note that the plotted points in the range of Fourier frequencies of 10 Hz to a few hundred Hz indicate a roll-off of approximately 1/f which is contrary to the usual power law process. This roll-off results from inadequate isolation of the two sources. Light injection locking of the two sources can therefore cause erroneous measurement results. The dotted line of the $1/f^3$ power law process indicates the probable response of the oscillators. The same roll-off and the dotted line projections are indicated on Figure 12.13d.

Figure 12.13b illustrates the action of a phase-lock loop in reducing the phase noise of the measurement system within the loop bandwidth of the VCXO. At the end of the phase-lock loop of the VCXO there is a roll-off and then the plotted data is the phase noise of the synthesizer. As indicated by the plot, the synthesizer has an internal phase-lock loop which reduces the phase noise out to the end of the loop at about 1.5 MHz. At the end of this phase-lock loop the characteristics of the synthesizer source oscillator are being measured. Normally the phase noise of the reference is attenuated by the loop attenuation characteristics outside the loop bandwidth. In this case, the reference noise is also being attenuated by the synthesizer phase-lock loop.

Figure 12.13c illustrates the type of phase noise characteristics that are measured when the reference source is phase-locked with an external loop. The oscillator characteristics are being measured outside the phase-lock loop bandwidth. Obviously, if one desires to measure the source closer to the carrier, the loop bandwidth must be narrower. This is a good example of phase noise reduction inside the loop bandwidth.

Figure 12.13d illustrates measurements of two identical 5 MHz signal sources which were multiplied using identical multiplier chains and the measurement of phase noise was made at the 100 MHz output frequency. The theoretical noise enhancement due to multiplication is (n^2 = 400 = 26.02 dB). The noise enhancement is slightly greater due to the added noise of the two multiplier chains. The phase noise level is shown as being reduced to the same level as the 5 MHz level by the use of a filter.

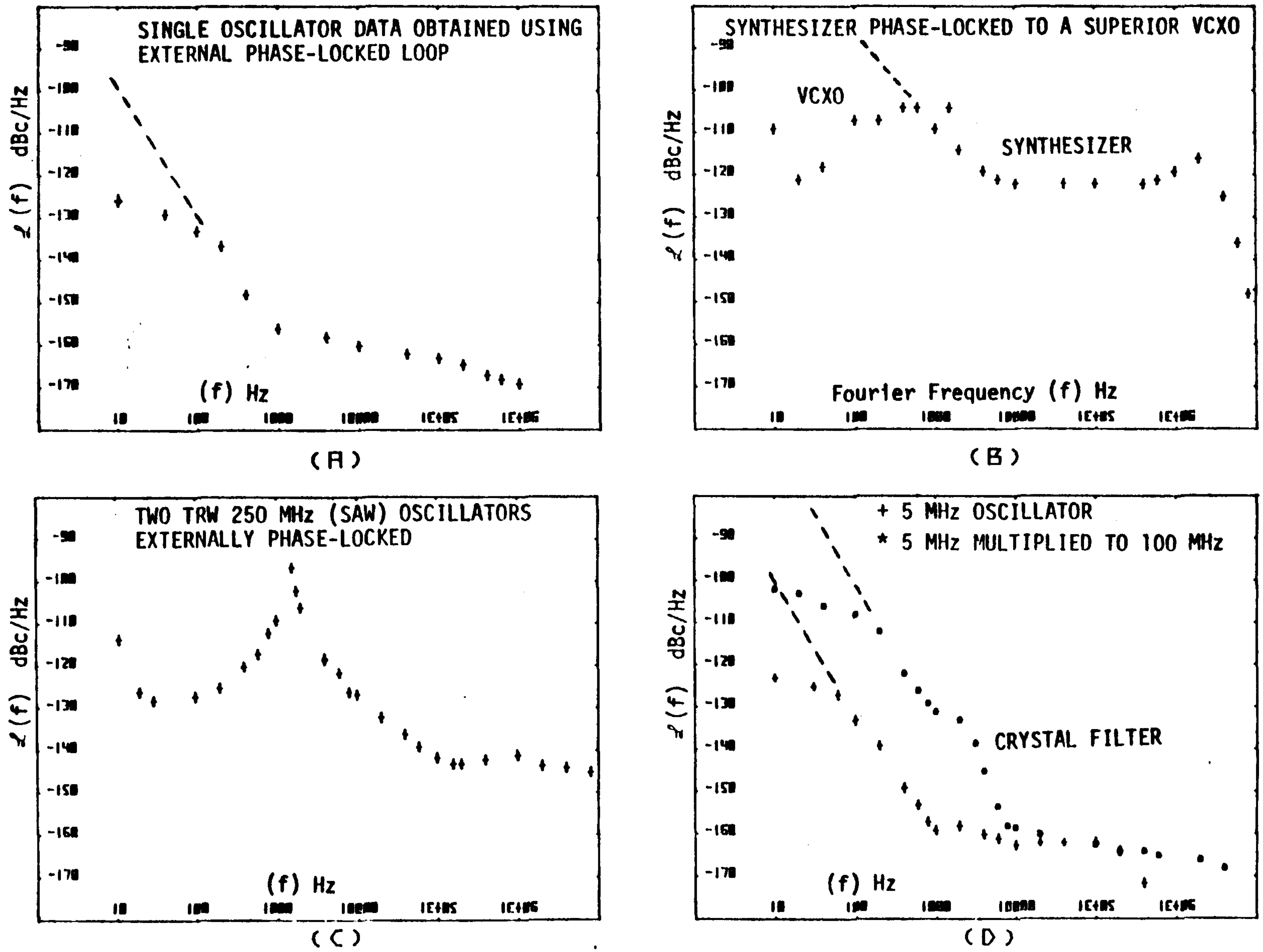

FIGURE 12.13 SAMPLE CALIBRATION DATA EXTRACTED FROM ACTUAL MEASUREMENTS

12.7 AUTOMATING PHASE NOISE MEASUREMENTS USING THE TWO-OSCILLATOR TECHNIQUE

The phase noise measurement system shown in Fig. 12.14 is program controlled by the Hewlett-Packard 9830 Calculator. Each step of the measurement process (calibration sequence and measurement mode) is included in the program. The measurements are not completely automated since the calibration sequence requires several manual operations. The baseband measurements are fully automated. The software program controls frequency selection, bandwidth settings, settling time, amplitude ranging, measurements, calculations and graphics and data plotting.

The system will be described as it is used to obtain a direct plot of the NBS defined spectral density Script $\mathcal{L}(f)$. The direct measurement of $\mathcal{L}(f)$ is represented by the following equation.

$$\mathcal{L}(f) = -\,[\text{Carrier Power Level} - (\text{Noise Power Level} - 6\text{ dB} + 2.5\text{ dB} - 10\log(B) - 3\text{ dB})]\ \text{dBc/Hz} \tag{12.59}$$

The noise power is measured relative to the carrier power level and the remaining terms of the equation represent corrections that must be applied due to the type of measurement and the characteristics of the measurement equipment as follows.

- The measurement of noise sidebands with the signals in phase quadrature requires the - 6 dB correction which is noted in Equation 16.
- The nonlinearity of the spectrum analyzer logarithmic IF amplifier results in compression of the noise peaks which, when average detected, require the + 2.5 dB correction.
- The bandwidth correction is required because the spectrum analyzer measurements of random or white noise are a function of the particular bandwidth used in the measurement.
- The - 3 dB correction is required since this is a direct measure of $\mathcal{L}(f)$ of <u>two oscillators</u>, assuming that the oscillators are of a similar type and that the noise contribution is the same for each oscillator. If one oscillator is sufficiently superior to the other, this correction is not required.

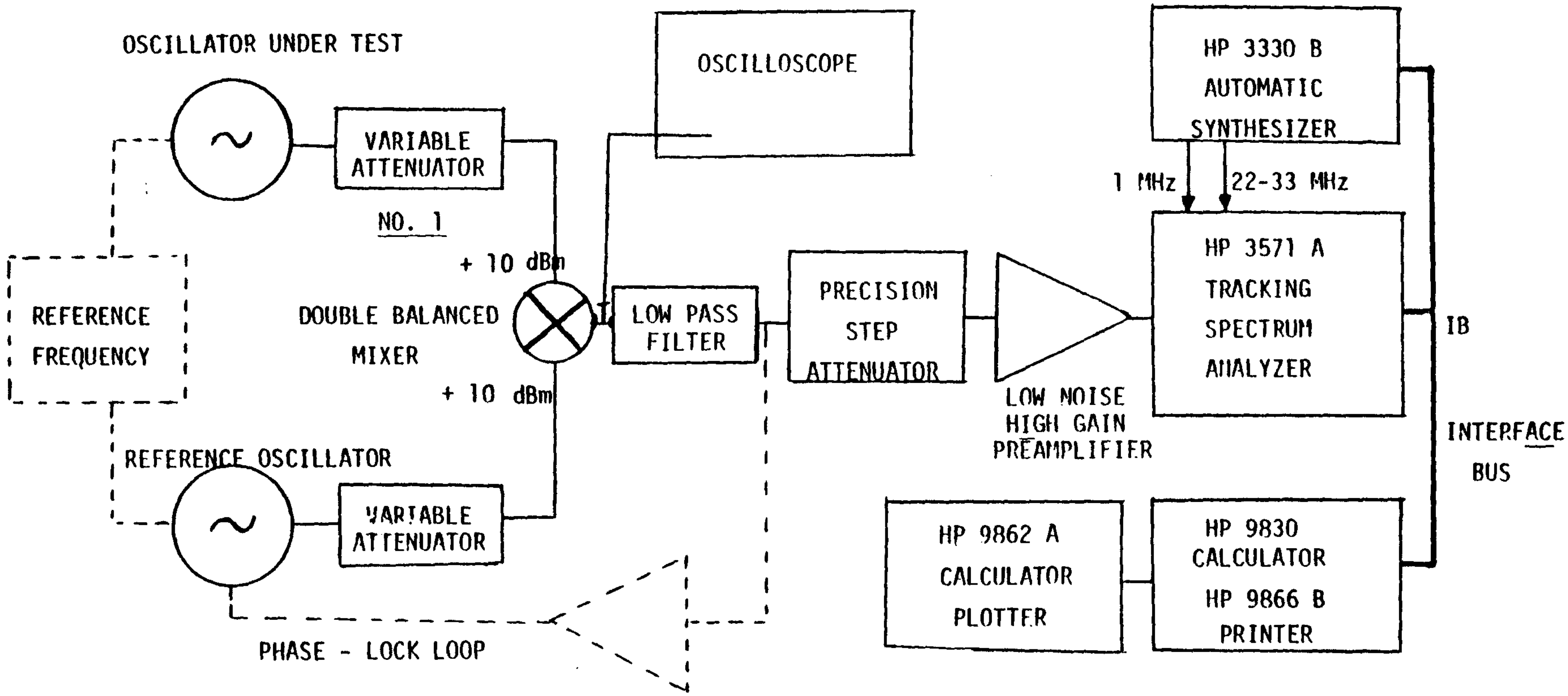

FIGURE 12.14. Basic Phase Noise Measurement System using the Two Oscillator Technique.

The system calibration and measurements are outlined as follows:

1. Measure the noise power bandwidth of each IF bandwidth setting on the Tracking Spectrum Analyzer.
2. Obtain a carrier power reference level (referenced to the output of the mixer).
3. Adjust for quadrature of the two signals applied to the mixer.

   Note: If the noise floor of the system has not been established, it is advisable to include a short program to check the phase noise versus system noise floor. This noise floor scan can be at a small number of points at selected Fourier frequencies over the measurement range.
4. Noise power is measured at the selected Fourier frequencies, the calculations are performed, and the data is plotted (or stored) using calculator and program control (fully automated).
5. Measure and plot the system noise floor characteristics if desired.

Measurement of Noise Power Bandwidth

The precise IF noise power bandwidth of the Tracking Spectrum Analyzer must be known when performing phase noise measurements. The basic automated measurement is as follows:

(a) A tee junction is inserted at the HP 3330B synthesizer 1 MHz output and the 1 MHz signal is applied to the input of the HP 3571A Tracking Spectrum Analyzer.

(b) The calculator controls the synthesizer for the desired incremental changes in frequency. The power output is recorded for each frequency setting over the range indicated in Figure 12.15. The range of measurements is illustrated for equal dB values on each side of the 1 MHz center frequency. One should choose points greater than 40dB below the carrier. We use 100 increments in frequency. Our experience has indicated that the 40dB level and the 100 increments in frequency are not the absolute minimum permissible values.

(c) The above recorded data is plotted for each IF bandwidth as illustrated in Figure 12.15. The noise power bandwidth is calculated as,

$$\text{Noise bandwidth (Hz)} = \frac{(P_1 + P_2 + P_3 + \text{---}P_{100})\Delta f}{\text{Peak Power Reading}} \tag{12.60}$$

where Δf is the frequency increment (Hz) and the peak power is the maximum measured point obtained during the measurements. All power values are in watts.

The resulting plot of Figure 12.15 can also be used to obtain the noise power bandwidth by numerical integration. The noise power bandwidth values are used in the calculations when performing the automated measurements.

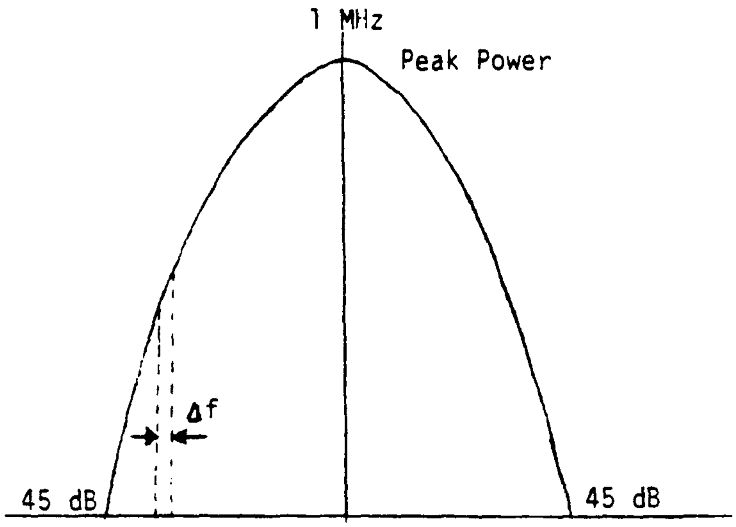

Figure 12.15. Illustration of a plot of measured noise power bandwidth.

Setting Carrier Power Reference Level

Essentially, the carrier reference power level is established at the output of the low-pass filter in Figure 12.14 as follows:

(a) The precision IF step attenuator is set to a high value to prevent overloading the spectrum analyzer (assume 50 dB as our example).

(b) Approximately equal power is applied to the inputs of the mixer as indicated in Figure 12.14.

(c) If the frequency of one of the oscillators can be adjusted, adjust its frequency for an IF output frequency in the range of 10 to 20 kHz. If neither oscillator is adjustable, replace the oscillator under test with one that can be adjusted as required and that can be set to the identical power level of the oscillator under test.

(d) The resulting IF power level is measured by the spectrum analyzer and the measured value is corrected for the attenuator setting which was assumed to be 50 db. The correction is necessary since this attenuator will be set to its Zero dB indication during the measurements of noise power. Assuming a spectrum analyzer reading of - 40 dBm, the carrier power reference level is calculated as,

$$\text{Carrier reference power level} = 50\text{ dB} - 40\text{ dBm} = 10\text{ dBm} \qquad (12.61)$$

Phase Quadrature Adjust

After the carrier power reference has been established, the oscillator under test and the reference oscillator are tuned to the same frequency and the original reference levels that were used during calibration are re-established. The quadrature adjustment depends upon the type of system used. Three possibilities are illustrated in Figure 12.14.

(a) If the oscillators are very stable, have high resolution tuning and are not phase-locked, the frequency of one oscillator is adjusted for zero dc voltage output of the mixer as indicated by the sensitive oscilloscope.

(b) If one oscillator is phase-locked using a phase-lock loop as shown dotted in on Figure 12.14, the frequency of the unit under test is adjusted for zero dc output of the mixer as indicated on the oscilloscope.

(c) If the common reference frequency is used, as illustrated in Figure 12.14, then it is necessary to include a phase shifter in the line between one of the oscillators and the mixer. (preferably between the attenuator and mixer). The phase shifter is adjusted to obtain zero dc output of the mixer.

NOTE: Throughout the measurement process one should check and maintain phase quadrature.

Measurements, Calculations and Data Plots

The measurement sequence is automated except for the case where manual adjustments are required to maintain phase quadrature of the signals.

Automated measurements are executed after phase quadrature is obtained and the Precision IF Step Attenuator is set to its Zero dB indication.

As an example, our previous assumptions resulted in a carrier power level of +10 dBm. Assume that the spectrum analyzer measures - 106 dBm with a 10 Hz bandwidth setting at a particular Fourier frequency. The value of spectral density which will be plotted or stored is calculated as,

$$\mathcal{L}(f) = -\,[10\text{ dBm} - (-106\text{ dBm/10Hz} - 6\text{ dB} + 2.5\text{ dB} - 10\log 10 - 3\text{ dB})] = -132.5\text{ dBc/Hz} \qquad (12.62)$$

The - 3 dB correction is included since it is assumed that the two units under test are similar and produce the same amount of noise.

The plotted or stored value of the spectral density of phase fluctuations in dB relative to one square radian is calculated as,

$$S_{\delta\phi}(f) = -132.5 + 3 \text{ dB} = -129.5 \text{ dBr} \qquad (12.63)$$

If one desires to plot the spectral density of frequency fluctuations, it is necessary to perform the following calculations,

$$S_{\delta\phi}(f) = 10^{(S_{\delta\phi}(f) \text{ in dBr})/10} \quad \text{radians}^2/\text{Hz} \qquad (12.64)$$

The spectral density of frequency fluctuations is,

$$S_{\delta\nu}(f) = f^2 \, S_{\delta\phi}(f) \quad \text{Hz}^2/\text{Hz} \qquad (12.65)$$

SYSTEM NOISE FLOOR VERIFICATION

A plot of the noise floor is obtained by repeating the automated measurements with Attenuator No. 1 set to maximum or by disconnecting the unit under test and terminating the input of the mixer with a matched load. The following equation is used when a correction for noise floor contribution is desired or necessary.

$$\mathcal{L}(f) \text{ (corrected)} = - \mathcal{L}(f) + 10 \log \left[\frac{P_{\mathcal{L}(f)} - P_{\text{noise floor}}}{P_{\mathcal{L}(f)}} \right] \text{ dBc/Hz} \qquad (12.66)$$

7.0 BASICS OF THE TRW METROLOGY ATUOMATED PHASE NOISE MEASUREMENT PROGRAM

(a) The HP 3330B Synthesizer serves as the local oscillator for the HP 3571A Tracking Spectrum Analyzer. The caluclator program controls the switching of the synthesizer to the desired offset frequencies.

(b) The offset frequency increments are chosen to be equal to the selected IF noise bandwidth in order to obtain a continuous spectrum plot.

(c) The minimum delay time for a measurement is determined by the IF filter build-up in the spectrum analyzer. The range is from 2.5 seconds for the 3 Hz bandwidth, decreasing to 70 milliseconds for the 10 kHz bandwidth setting.

(d) The IF bandwidth settings for the Fourier (offset) frequency range selections are as follows:

IF BANDWIDTH	FOURIER FREQUENCY
3 Hz	10 Hz to 400 Hz
10 Hz	400 Hz to 1 kHz
30 Hz	1 kHz to 4 kHz
100 Hz	4 kHz to 10 kHz
200 Hz	10 kHz to 40 kHz
1 kHz	40 kHz to 100 kHz
3 kHz	100 kHz to 400 kHz
10 kHz	400 kHz to 13 MHz

Program running time is 23 minutes.

(e) Video smoothing is used in order to obtain a better approximation of the mean. The program can be designed so that a large number of measurements can be taken for better estimation of the mean value.

(f) The 60 Hz line frequency interference appears smaller than the actual amplitude if the noise corrections are applied as set forth in the noise measurement program. The corrections for the log amplifier and detection, bandwidth and equal oscillator contribution should be removed for a plot of discrete frequencies.

(g) Amplitude auto-ranging is used in the program to select the most sensitive range that does not result in overload conditions.

The low-pass filter prevents local oscillator leakage power from overloading the spectrum analyzer when baseband measurements are performed at the Fourier (offset) frequencies of interest. Leakage signals will interfere with autoranging and the dynamic range of the spectrum analyzer.

The low-noise high-gain preamplifier provides additional system sensitivity by amplifying the noise signals to be measured. Also, since spectrum analyzers usually have high values of noise figure, this amplifier is highly desirable. As an example, if the high-gain preamplifier had a noise figure of 3dB and the spectrum analyzer had a noise figure of 18dB, the system sensitivity at this point has been improved by 15dB. The overall system sensitivity would not necessarily be improved 15dB in all cases because the limiting sensitivity could have been imposed by a noisy mixer.

Figure 12.16 shows the continuous plot obtained using the two-oscillator technique and the TRW Metrology automated program. The 60 Hz signal and line harmonics were not removed or avoided. Also, the radio signals are present since the measurements were not measured in a screen room. The amplitude of these signals is greater than shown on the plot since the noise corrections were not removed.

The roll-off characteristic is again noted in the range of Fourier frequency from 10 Hz out to a few hundred hertz. Slight injection locking was noted during the measurement. This measurement system characteristic does not occur if the two sources are locked to a common source as illustrated in Figure 12.14. The phase noise plot of these oscillators showed a $1/f^3$ power law process from the 10 Hz point when measured in two different delay line FM discriminator measurement systems.

If the oscillator under test has low noise (lower than -150 dBc/Hz) at a Fourier frequency of 1 MHz, there will be a noticeable positive peaking of the phase noise plot. This is caused by the second LO of the Tracking Spectrum Analyzer. A plot of the system noise floor will show this characteristic.

The phase noise characteristics of a synthesizer are plotted in Figure 12.17. The measurements were performed using the two-oscillator technique with the two synthesizers locked to a common source as shown by the dotted lines in Figure 12.14. Significant points are shown on the plot.

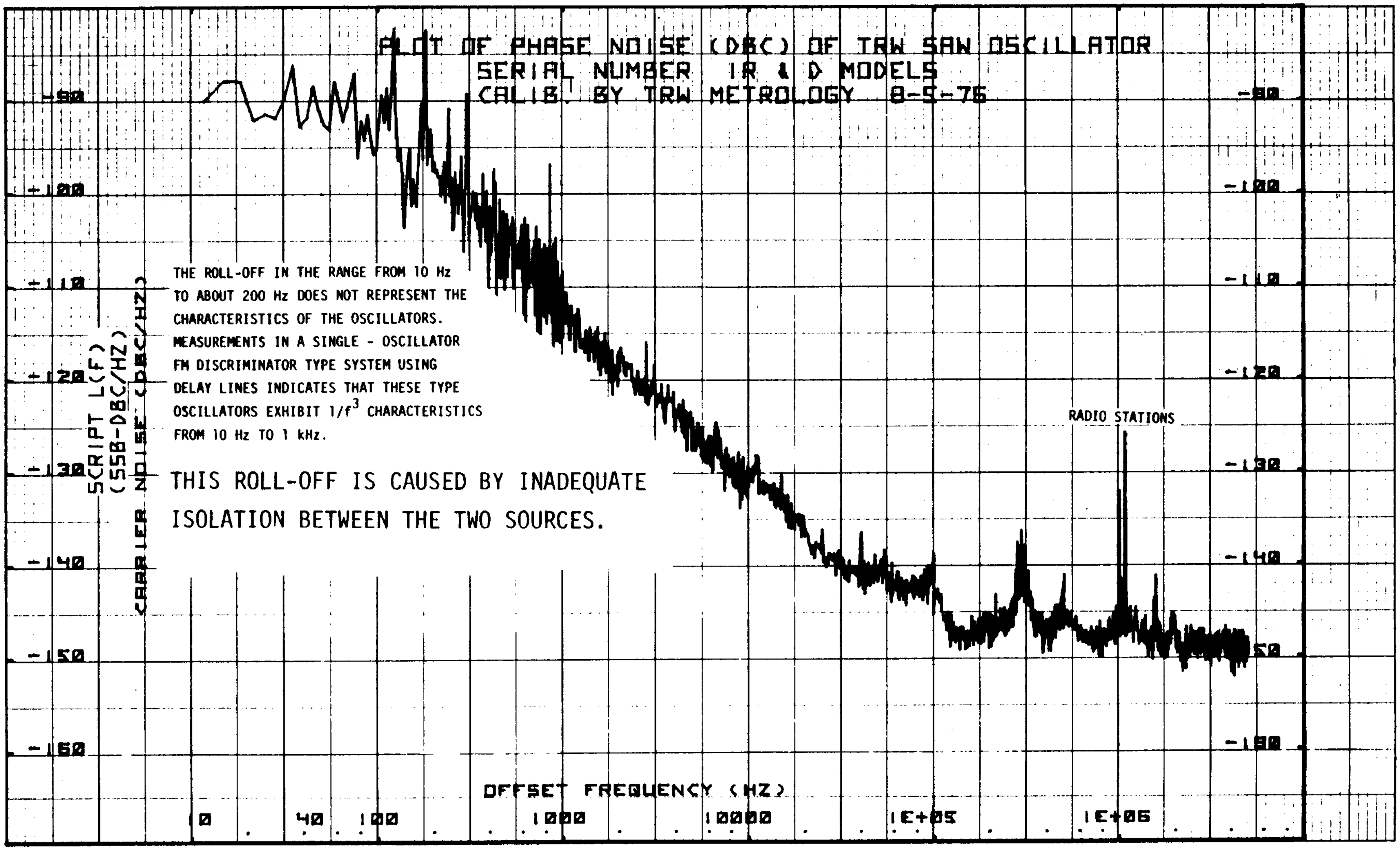

FIGURE 12.16. PHASE NOISE PLOT USING THE TWO-OSCILLATOR TECHNIQUE.

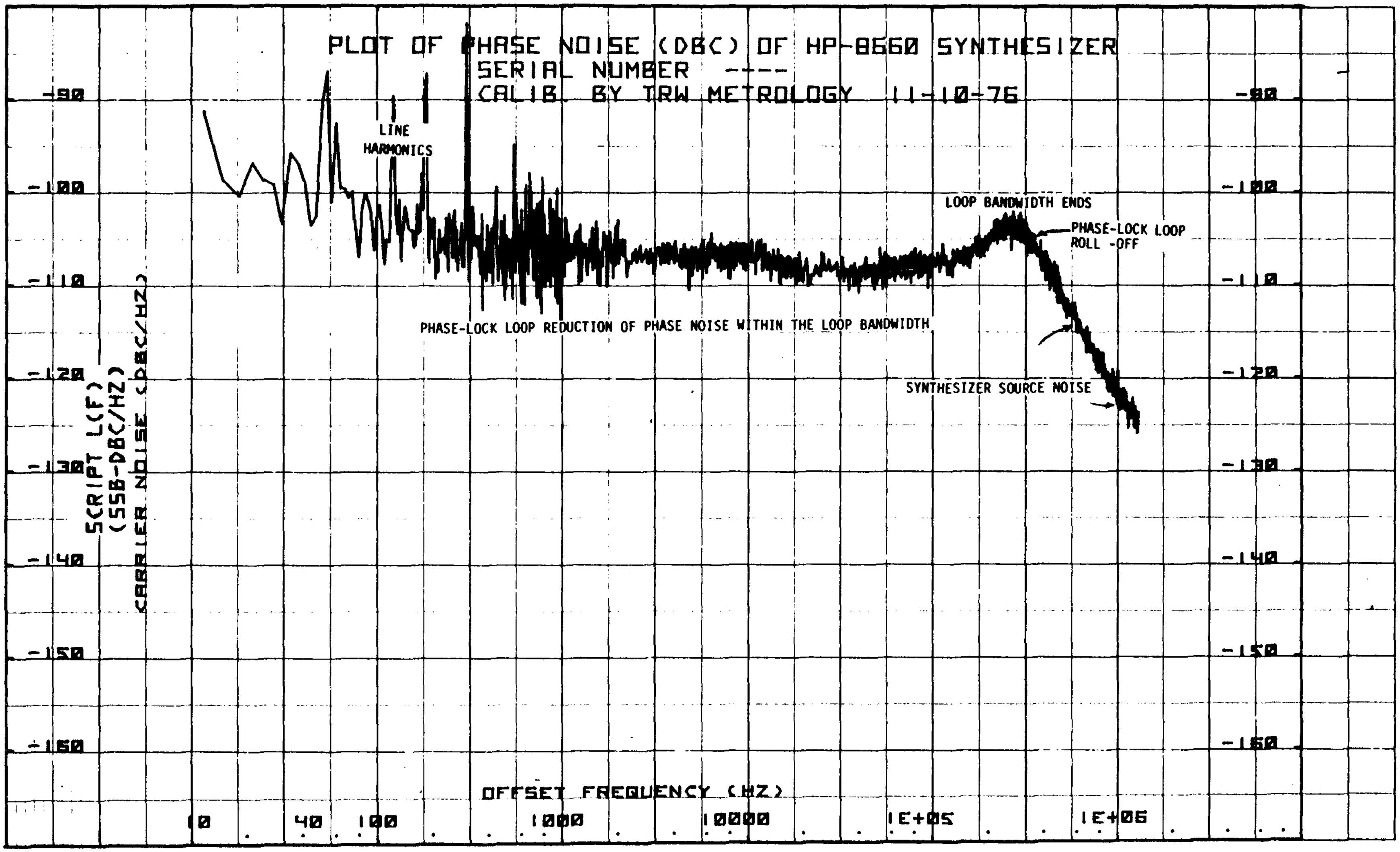

FIGURE 12.17. PHASE NOISE PLOT OF A SYNTHESIZER USING THE TWO-OSCILLATOR TECHNIQUE.

12.8 PHASE NOISE MEASUREMENTS USING DELAY LINES AS FM DISCRIMINATORS

Frequency fluctuations are measured using discriminator techniques. One of the important advantages of this type system is that the phase noise characteristics of a single oscillator can be measured without the requirement of a similar or better source as a reference.

The delay line yields a phase shift by the time the signal arrives at the balanced mixer. The phase shift depends upon the instantaneous frequency of the signal. The presence of frequency modulation (FM) on the signal gives rise to differential phase modulation (PM) at the output of the differential delay and its associated (non-delay) reference line. This relationship is linear if the delay (τ_d) is non-dispersive. This is the property which allows the delay line to be used as an FM discriminator. In general, the conversion factors are a function of the delay (τ_d) and the Fourier frequency (f) but not the carrier frequency. The delay of the transmission line and variable length airline as a function of phase noise measurement at various offset frequencies will be discussed later. However, the maximum sensitivity of the transmission line discriminator depends upon the attenuation value. For maximum sensitivity, the optimum length of the line for the system in Figure 12.18, (Ashley et al,[25]), occurs when the total attenuation is one neper (8.686dB). In this system it represents the two-way or round trip loss of the delay line. The optimum delay line length is determined at a particular selectable frequency. However, since the attenuation varies slowly (approximately proportional to the square root of frequency), this characteristic allows near optimum operation over a considerable frequency range without appreciable degradation in the measurements. Two distinctly different delay line discriminators will be described in this presentation. The first system, developed by Ashely et al [25], is shown in Figure 12.18.

The single oscillator signal is split into two channels in the system. One channel is called the non-delay or reference channel. It is also referred to as the local oscillator channel since the signal in this channel drives the mixer at the prescribed impedance level (the usual LO drive). The signal in the second channel arrives at the mixer through a delay line.

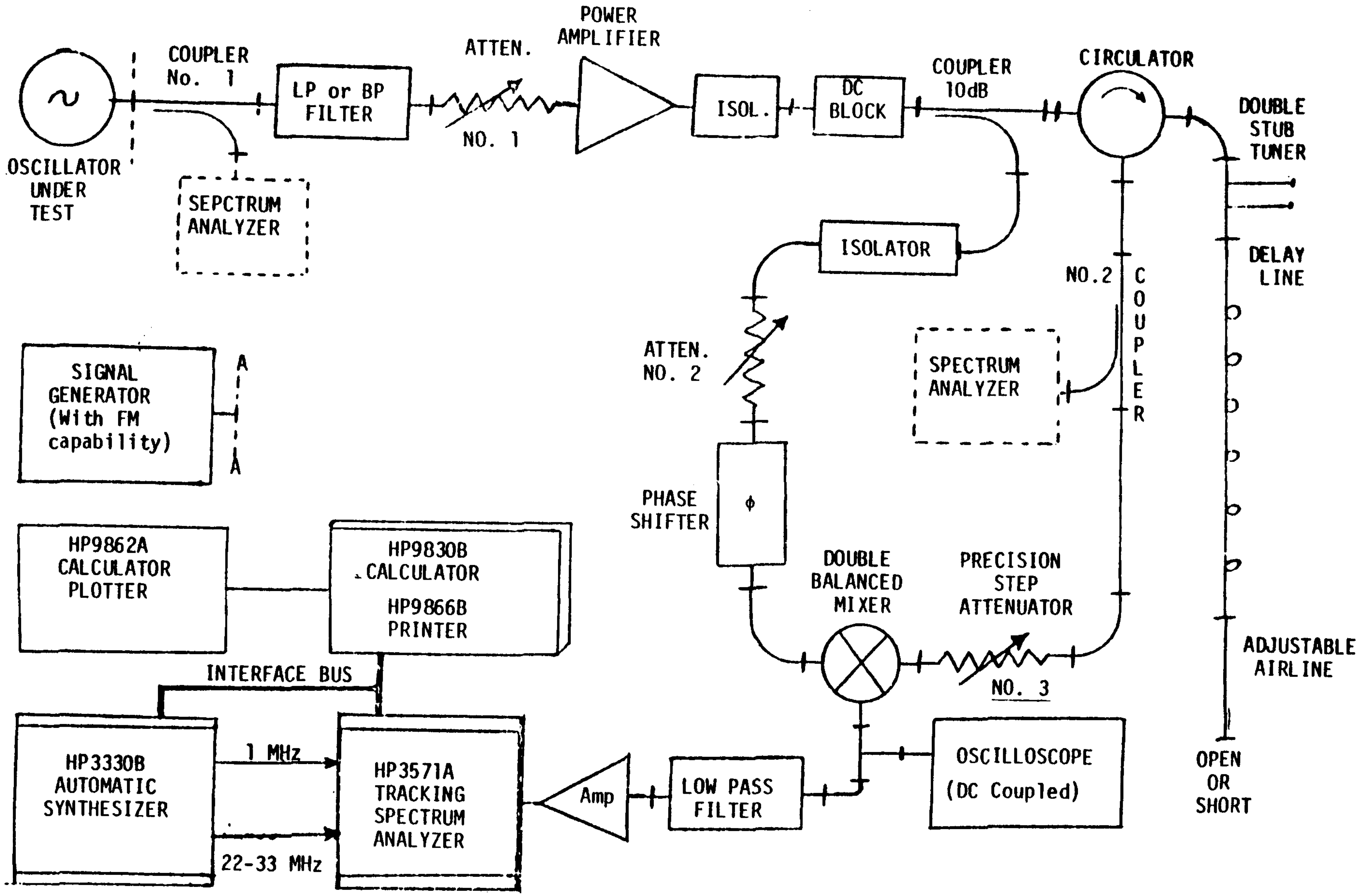

FIGURE 12.18. Single Oscillator Phase Noise Measurement System Using the Circulator/Delay Line FM Discriminator.

The circulator, tuner, delay line and adjustable airline form a discriminator which is tuned to suppress (null out) the carrier. Since the carrier is suppressed, the system is not restricted by burnout characteristics of the crystals in the phase detector (mixer).

The carrier nulling procedure makes the discriminator relatively immune to incidental AM on the signal being tested. The ideal carrier suppression filter would provide infinite attenuation of the carrier and zero attenuation of all other frequencies. The practical discriminator, used as a filter, has finite bandwidth that suppresses the carrier and the sidebands on both sides of the carrier. The effective Q of the discriminator determines how much the signals are attenuated.

Since the carrier is nulled out, a high-gain power amplifier is used to increase the amplitude of the noise signals applied to the mixer. This amplifier must not contribute additional noise to the signal. The noise contribution can be checked when one has two identical power amplifiers.

The basic steps of the measurement procedure are as follows:

(a) Measure the IF noise power bandwidth of each IF bandwidth position of the spectrum analyzer to be used in the measurement process.

The measurement technique was described in the previous calibrations associated with the measurement system of Figure 12.14.

(b) Establish the system power levels.

(c) Null out the carrier.

(d) Calibrate the discriminator.

(e) Measure and plot the oscillator characteristics in the automatic measurement mode.

(f) Measure the system noise floor.

NOTE: It is not necessary to measure the carrier power reference level when using the discriminator type measurement systems.

SYSTEM POWER LEVELS

The system power levels are set for maximum power applied to the discriminator and appropriate power applied to the mixer through the non-delay channel.

(a) Set Attenuators 1 and 2 to maximum attenuation and set Attenuator No. 3 to 50 dB.
Attenuator No. 3 is set to 50 dB in order to prevent damage or overloading of the mixer during the carrier nulling procedure. This attenuator will be set to Zero dB when the measurements of frequency fluctuations are performed.

(b) With all equipment operating properly, Attenuator No. 1 is used to set the system power level. Unless limited by some characteristic of the amplifier, this attenuator will be set to zero attenuation so that maximum power will be applied to the discriminator.

(c) Attenuator No. 2, in the reference channel, is now adjusted to apply the proper local oscillator drive level to the mixer.

CARRIER NULL

Adjust the double stub tuner and airline to null out the carrier as observed on the spectrum analyzer connected to Coupler No. 2. This null is monitored throughout the complete measurement sequence. If the null is not maintained at any point in the program, the measurements are stopped and the carrier is renulled.

DISCRIMINATOR CALIBRATION

The calibration factor of the discriminator should be established at selected Fourier frequencies over the range of measurements. If the calibration factor is not constant, appropriate measured values should be used in the automated measurement program. The measurement and calibration will be described in terms of a 20 kHz modulation frequency.

The calibration factor is defined as:

$$CF = \frac{\Delta\nu_{rms}}{V_{rms}} \quad \text{Hz/volt} \tag{12.67}$$

Where $\Delta\nu_{rms}$ is the rms frequency deviation of the carrier due to the intentional modulation and V_{rms} is the spectrum analyzer voltage measurement of the modulation sideband (Fourier frequency).

$$\Delta\nu_{rms} = \frac{\Delta\nu_{peak}}{\sqrt{2}} = \frac{m\ (f_m)}{\sqrt{2}} \quad \text{Hz/volt} \qquad (12.68)$$

The calibration factor of the discriminator is calculated as:

$$CF = \frac{m\ (f_m)}{\sqrt{2}\ V_{rms}} \quad \text{Hz/volt} \qquad (12.69)$$

The calibration factor is established as follows:

(a) Replace the oscillator under test with a signal generator or oscillator that can be frequency modulated. The power output and operating frequency of the generator must be set to the same precise frequency and amplitude values as the oscillator under test.

(b) Select a modulation frequency of 20 kHz and increase the modulation until the carrier is reduced to the first Bessel null as indicated on the spectrum analyzer connected to Coupler No. 1. One can choose any other convenient setting that will produce a known modulation index (m).

(c) Tune the HP 3571A Tracking Spectrum Analyzer to the modulation sideband frequency (20 kHz) and adjust the phase shifter for maximum output power as indicated on the spectrum analyzer. This sets the phase quadrature of the two signals applied to the balanced mixer.

(d) The Tracking Spectrum Analyzer is now displaying a power reading that corresponds to the modulation frequency. This reading must be corrected for the dB setting of Attenuator No. 3 in order to calculate the calibration factor of the discriminator. Attenuator No. 3 will be set to Zero dB reference indication during the measurements of Fourier frequency noise power. This results in an increase of sideband power at the mixer which must be accounted for in the calculation of the calibration factor.

As an example, if the attenuator was set to 50 dB as proposed, and the spectrum analyzer reading of baseband power is -35 dBm, the corrected power is then,

$$P\ (\text{dBm}) = 50\ \text{dB} - 35\ \text{dBm} = +15\ \text{dBm} \qquad (12.70)$$

(e) The discriminator calibration factor can now be calculated since this power in dBm can be converted to the corresponding rms voltage using the following equation.

$$V_{rms} = \sqrt{\frac{10^{P(dBm)/10}}{1000} \times R} \qquad (12.71)$$

Where R is 50 ohms in this system.

(f) The discriminator calibration factor is calculated as,

$$CF = \frac{m \cdot f_m}{\sqrt{2}\, V_{rms}} = \frac{2.405 \cdot f_m}{\sqrt{2}\, V_{rms}} \quad \text{Hz/volt} \qquad (12.72)$$

Since 2.405 is the modulation index (m) for the first Bessel null as used in this technique. The modulation frequency is f_m.

MEASUREMENT AND DATA PLOTTING

(a) Connect the unit under test in place of the modulated signal generator and readjust the carrier null if necessary.
(b) Set Attenuator No. 3 to its Zero dB indication.

The measurements, calculations and data plotting are completely automated. The calculator program selects the Fourier frequency, auto-ranging is performed, bandwidth is set, and measurements of Fourier frequency power are performed by the tracking spectrum analyzer.

Each Fourier frequency noise power reading P_n(dBm) is converted to the corresponding rms voltage designated as v_{1rms}.

$$v_{1rms} = \sqrt{\frac{10^{(P_n + 2.5)/10}}{1000} \times R} \qquad (12.73)$$

The rms frequency fluctuations are calculated as,

$$\delta\nu_{rms} = v_{1rms} \times CF \qquad (12.74)$$

The spectral density of frequency fluctuations is calculated as,

$$S_{\delta\nu}(f) = \frac{(\delta\nu_{rms})^2}{B} \quad \text{Hz}^2/\text{Hz} \qquad (12.75)$$

Where B is the measured IF noise power bandwidth of the spectrum analyzer.

The spectral density of phase fluctuations is calculated as,

$$S_{\delta\phi}(f) = \frac{S_{\delta\nu}(f)}{f^2} \quad [\text{radians}^2/\text{Hz}] \qquad (12.76)$$

The NBS designated spectral density is calculated as,

$$\mathcal{L}(f)_{dB} = 10 \log \frac{S_{\delta\phi}(f)}{2} \quad [\text{dBc/Hz}] \qquad (12.77)$$

The above spectral density is plotted in real time in our program. However, the data can be stored and the desired spectral density can be plotted in other forms.

NOISE FLOOR MEASUREMENTS

The system noise floor can be plotted by setting Attenuator No. 3 to maximum and repeating the automated measurements. In the calculation process in the program, the rms voltage corresponding to the noise floor is designated v_{2rms}.

In the TRW program the noise floor is only checked at selected Fourier frequencies over the range of measurement.

A correction for the noise floor requires a measurement of the rms voltage of the oscillator (v_{1rms}) and a measurement of the noise floor rms voltage (v_{2rms}). These voltages are used in the following equation to obtain the corrected value.

$$v_{rms} = \sqrt{(v_{1rms})^2 - (v_{2rms})^2} \qquad (12.78)$$

The value v_{rms} is then used in the calculation of frequency fluctuations. If adequate memory is available, each value of v_{1rms} can be stored and used after the other set of measurements are performed at the same Fourier frequencies.

The phase noise plot of Figure 12.19 was obtained with the TRW Metrology program as set forth in this presentation.

The phase noise plot follows the $1/f^2$ power law process from 1 kHz to about 400 kHz. Beyond this area the plot is periodic in $\omega = 2\pi f_d$, where f_d is the Fourier frequency corresponding to $1/t_d$, and t_d is the total (round-trip)

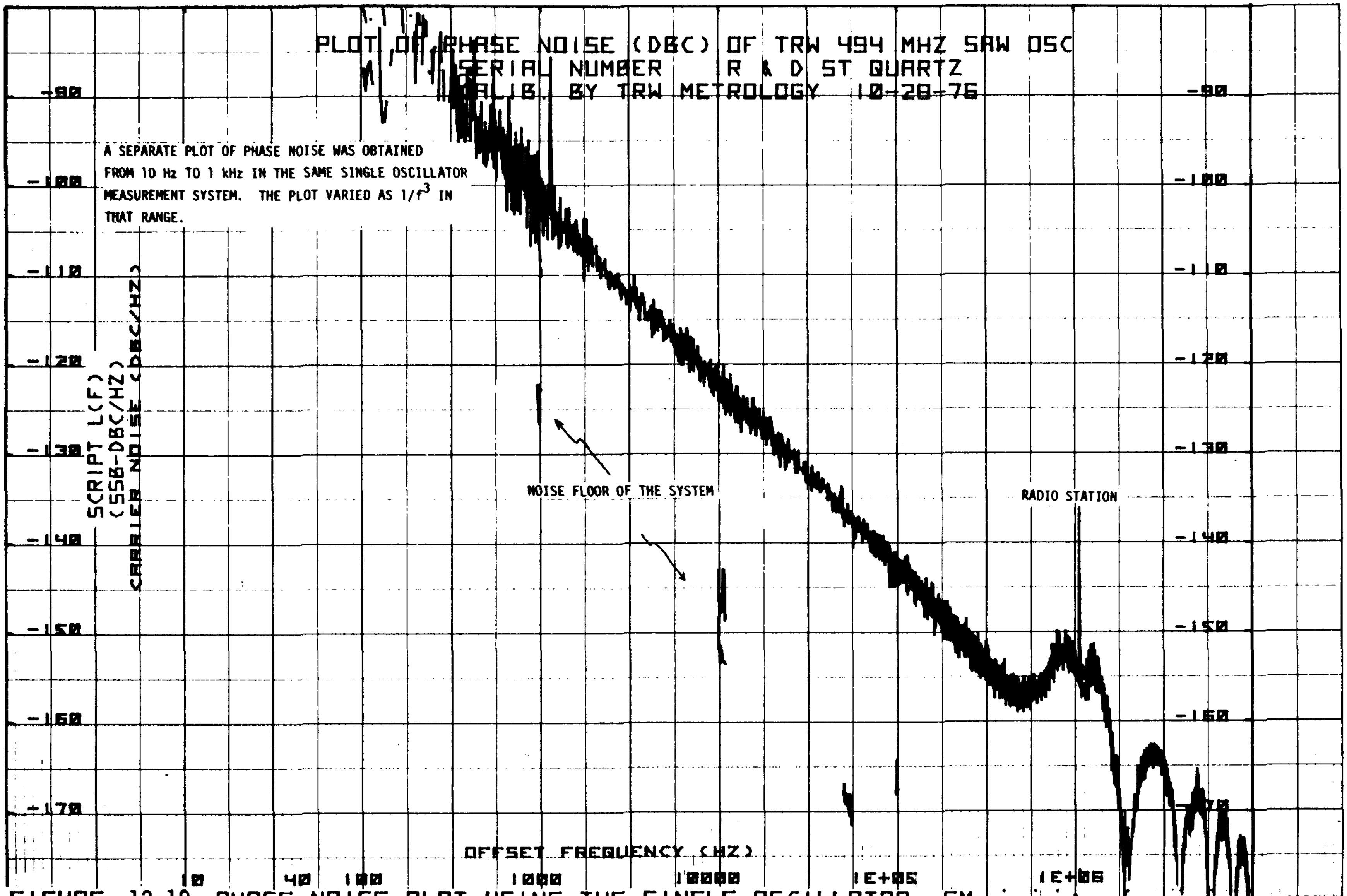

FIGURE 12.19. PHASE NOISE PLOT USING THE SINGLE-OSCILLATOR, FM DISCRIMINATOR DELAY LINE TECHNIQUE.

The period of the angular frequency (ω) is $t_d/2\pi$ or 500 nanoseconds per 2π radians or 360 degrees. i.e., the delay line is 250 ns in length.

The cyclic frequency period is $1/t_d$. Based on the 500 ns line, the first null of the plot is at $f_d = 1/t_d = 2$ MHz. The nulls are therefore spaced at 2 MHz intervals.

The data plot also indicates the relationships involving time delay and the measurement range in terms of the Fourier frequency.

The periodic display of plotted data for a 1000 ns delay line would show a peak at 400 kHz and a null at 1 MHz and at 1 MHz intervals on out.

Alternatively, a delay line of 250 ns would have produced a first peak at 2 MHz and the first null at 4 MHz.

This periodic display should be familiar to anyone who has experience in swept frequency techniques involving time delay measurements in single or dual channel systems.

This is not a precise measure of the time delay of the delay line. However, it is sufficiently accurate for these approximation purposes.

The phase noise of this oscillator was plotted on an expanded graph over the Fourier frequency range of 10 Hz to 1kHz where it exhibited a $1/f^3$ characteristic.

12.9 PHASE NOISE MEASUREMENT SYSTEM USING THE DELAY LINE DISCRIMINATOR WITHOUT CARRIER SUPPRESSION

The measurement system block diagram is shown in Figure 12.20. The signals in the delay line channel of the system experience the one-way delay of the line. For a given cable, the delay line in this system must be twice as long as the cable required in the system of Figure 12.14 in order to obtain the same time delay and one neper of attenuation at the same operating frequency.

With adequate drive power we have not limited the delay line in this system to one neper (8.686 dB). Delay lines with attenuation greater than 11 dB were used in some of our measurements. The resulting noise floor was more than 10 dB below the measured values of phase noise.

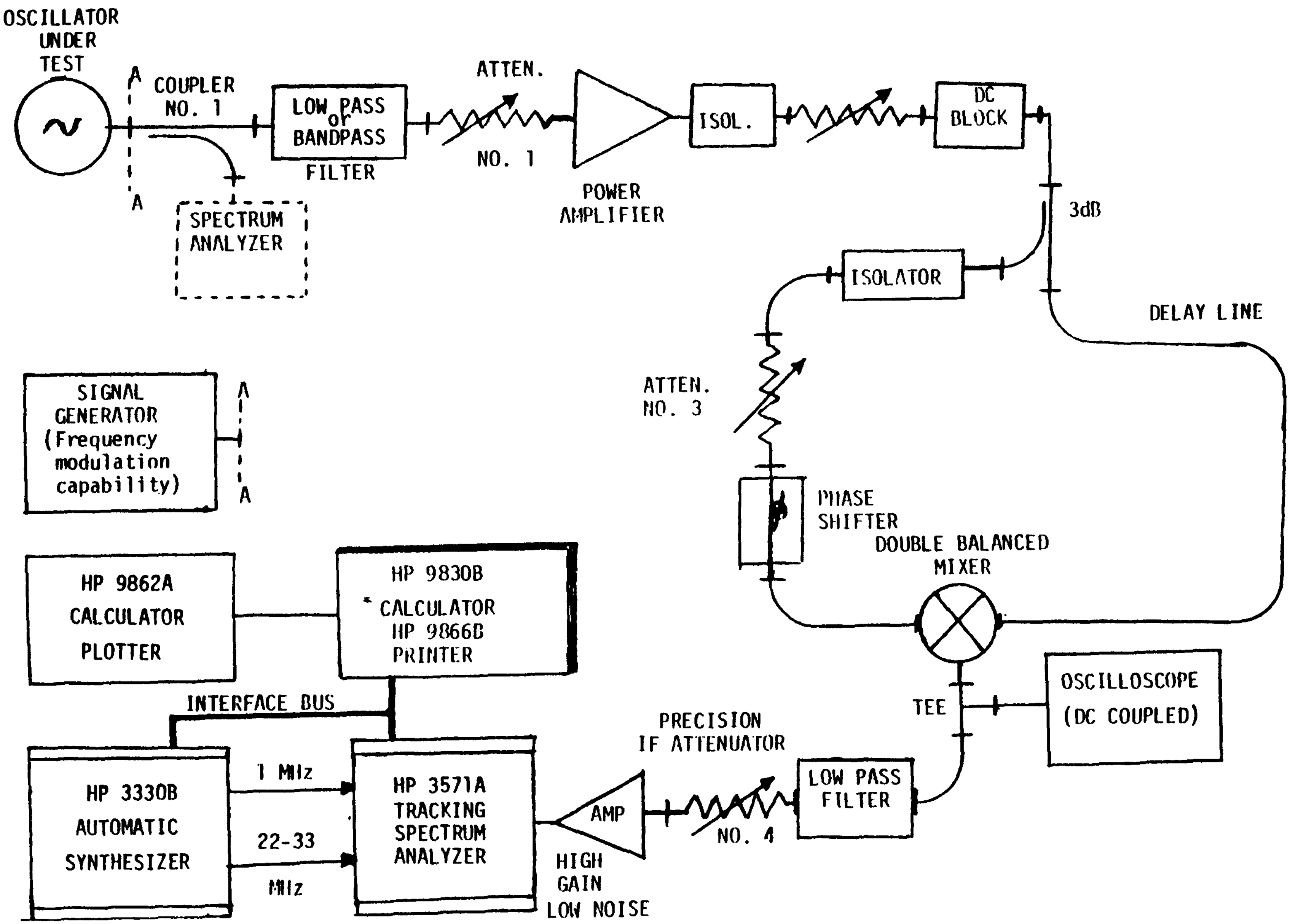

FIGURE 12.20. Phase Noise Measurement System Using t[illegible] M Discriminator Delay Line Without Carrier Suppression.

The system calibration is outlined as follows:

(a) The IF noise power bandwidth is measured for each IF bandwidth setting of the spectrum analyzer as set forth in a previous section of this paper.

(b) The oscillator under test is connected and Attenuators 1, 2, and 3 are used to set the equal power levels at the mixer inputs. The inputs are + 10 dBm for the particular mixer used in our system.

(c) The phase shifter is adjusted for Zero volts dc at the output of the mixer as indicated on the oscilloscope connected as shown. This establishes the quadrature condition for the two inputs to the mixer. This quadrature condition is continuously monitored and is adjusted if necessary.

(d) Attenuator No. 4 is set to 50 dB and the oscillator under test is replaced with a source that can be frequency modulated.

The modulated source frequency must be set to the precise frequency of the unit under test. The output power level must also be set to the precise output level of the unit under test.

(e) Modulate the source with a frequency of 10 or 20 kHz. Apply modulation to obtain the first Bessel null of the carrier as noted on the spectrum analyzer connected to Coupler No.1. This produces a modulation index of 2.405.

(f) Tune the Tracking Spectrum Analyzer to the modulation frequency as chosen in (e) above.

The power reading at this frequency is recorded in the program and is corrected for the 50 dB setting of Attenuator No. 4 which will be set to Zero dB indication during the automated measurements.

$$P(\text{dBm}) = (-\text{ dBm power reading}) + 50\text{ dB} \qquad (12.79)$$

The above power level is converted to the equivalent rms voltage that the spectrum analyzer would have read if the total signal had been applied.

$$V_{rms} = \sqrt{\frac{10^{P(\text{dBm})/10}}{1000} + R} \qquad (12.80)$$

The discriminator calibration factor is calculated as,

$$CF = \frac{2.405 \cdot f_m}{\sqrt{2}\, V_{rms}} \quad \text{Hz/volt} \qquad (12.81)$$

NOTE: The measured calibration factor in this system has been constant from a few kHz out to 200 kHz.

The automated measurements are performed and the data plots are established using the same measurements and equations as set forth for the previous system using the delay line as an FM discriminator.

The phase noise plots in Figure 12.21 show a plot of the fundamental oscillator at 600 MHz and a plot of the multiplied output at 2.4 GHz. The difference in lengths of the delay lines is indicated by the difference in the first nulls. It is noted that the measured and theoretical noise enhancement agree within the resolution of the plot. These measurements were performed using the system shown in Figure 12.20.

Identical phase noise plots were obtained using the two FM discriminator systems. Phase noise measurements, performed using the two-oscillator technique, were in very close agreement in the range from 1 kHz to the 400 kHz area. The disagreement in the range from 10 Hz to 1 kHz is due to the inadequate isolation when using the two-oscillator technique as previously explained.

Dr. J. Robert Ashley and Gustaf J. Rast, Jr. [25] performed measurements on a 250 MHz TRW Surface Acoustic Wave oscillator using their system at Redstone Arsenal. The measurement data was in close agreement with the data obtained using the three systems described in this paper.

Measurements, using two identical stable sources, were performed using the TRW program. The measurements were repeated using the Hewlett-Packard program in Application Note No. 207 and these plotted points fell precisely on the continuous plot.

The straight through delay line system of Figure 12.20 is more simple in construction and is easier to calibrate since the adjustments are less critical. Also, the calibration factor has been found to be constant, within the resolution of our measurement capability, out to several hundred kHz where limited by modulation capability.

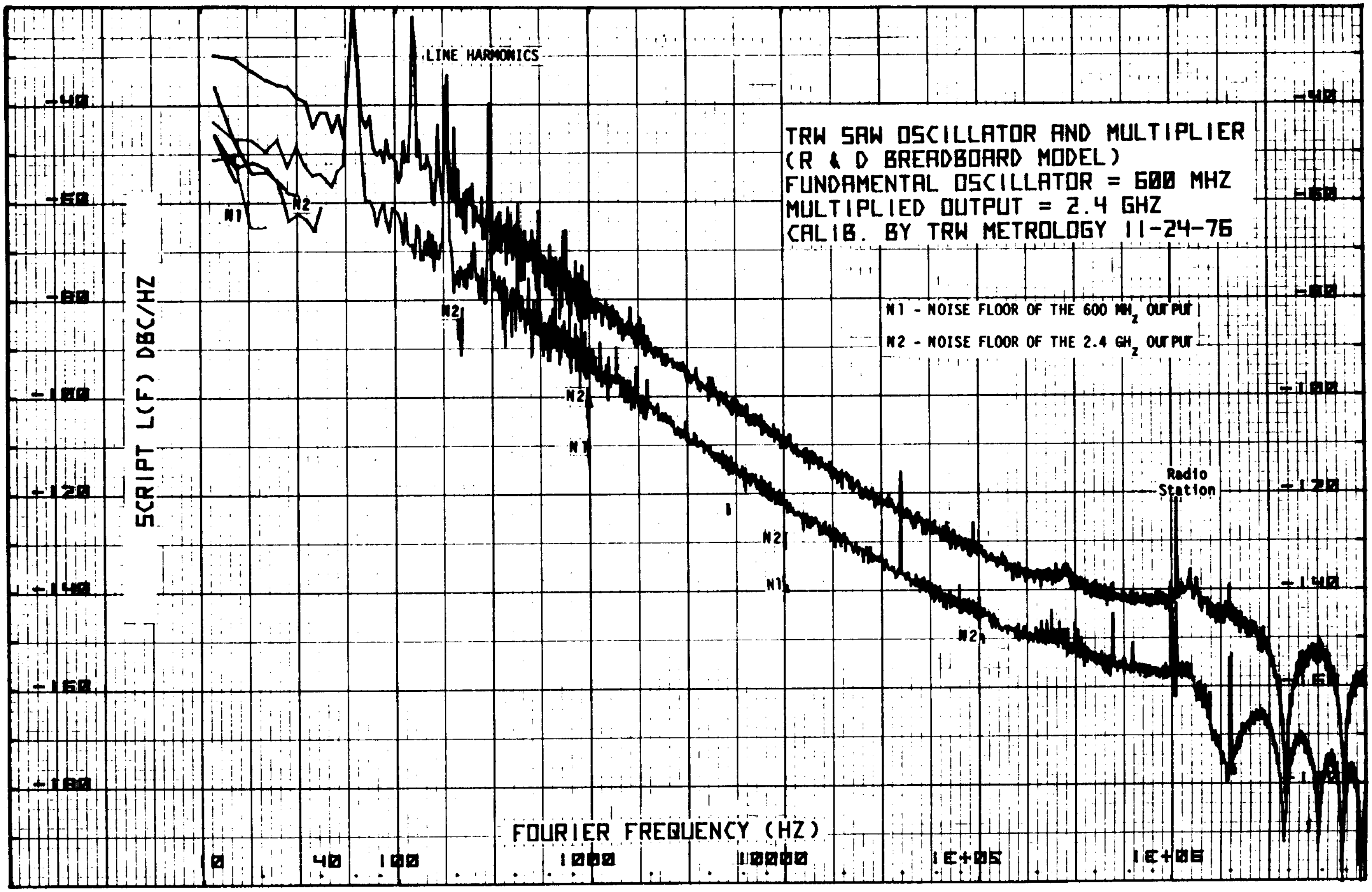

FIGURE 12.21. PHASE NOISE OF 600 MHZ OSCILLATOR MULTIPLIED TO 2.4 GHZ

12.10 PHASE NOISE MEASUREMENTS IN THE TIME DOMAIN

The parameters involved in the measurement of frequency fluctuations in the time domain are illustrated in Figure 12.22.

T - time interval between two successive measurements
τ - sampling time interval
τ_d - dead time interval $(T - \tau)$
t - time variable
t_o - arbitrary, fixed instant of time
t_k - the time coordinate of the beinning of the k-th measurement of average frequency . By definition $t_{k+1} = t_k + T$, $k = 0,1,2$ ---.

A sequence of measurements of frequency is performed and standard diviation (σ) is calculated using the calculated mean frequency (y_k) for the sequence. The problem with standard deviation is that change in measurement time, dead time, size of sample and bandwidth often affects the value of standard deviation.

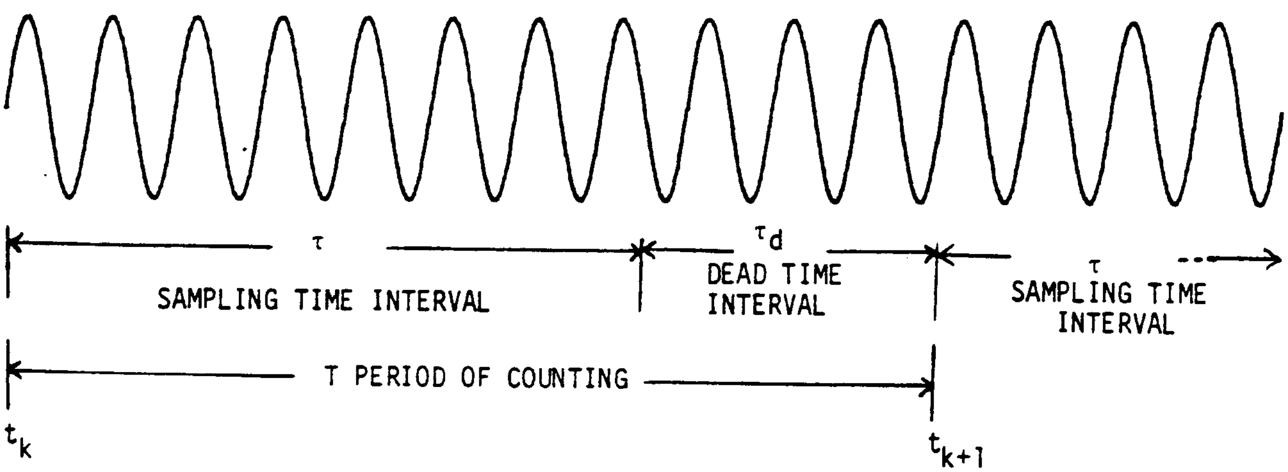

FIGURE 12.22 ILLUSTRATION OF TIME DOMAIN MEASUREMENT PARAMETERS.

Heterodyne Method

The heterodyne method for measuring frequency and frequency stability is illustrated in Figure 12.23. The frequency deviation from one instant to the next can be measured by measuring the beat frequency (ν_b). One hertz variation at the beat frequency corresponds to one hertz variation at the unknown oscillator frequency (ν_1).

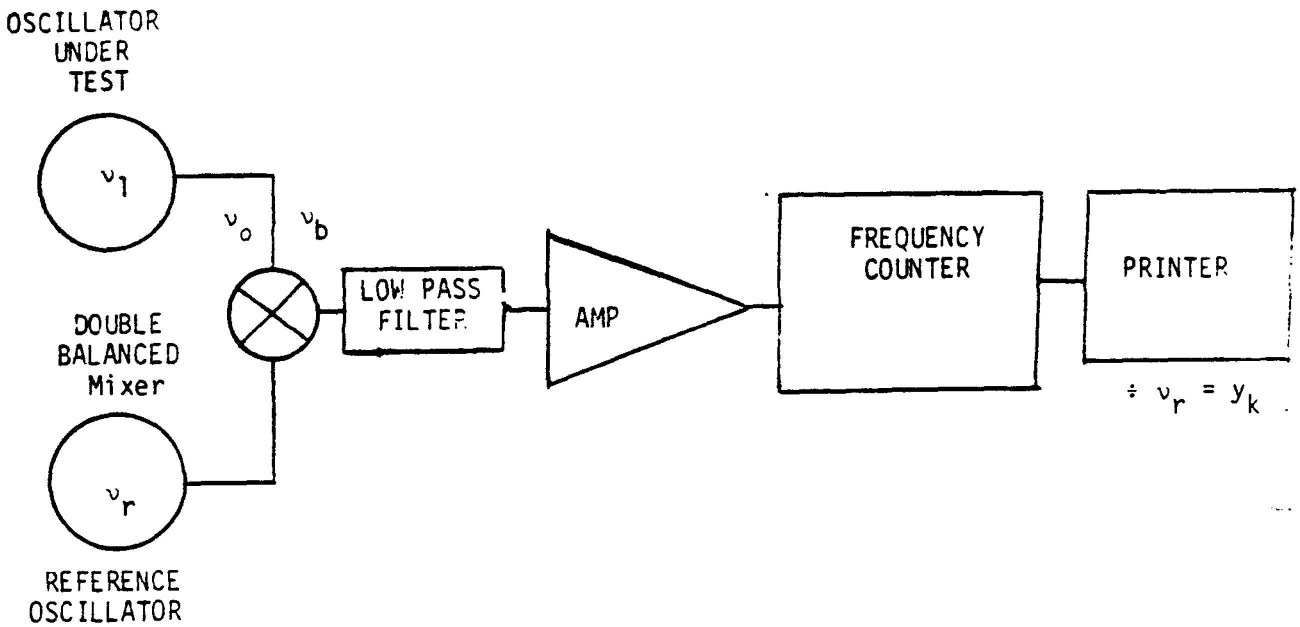

Figure 12.23 Heterodyne Method of Measuring Frequency and Frequency Stability.

The frequency offset (beat frequency) is defined by,

$$\nu_b = \nu_1 - \nu_r \tag{12.82}$$

The fractional frequency offset is,

$$\frac{\nu_b}{\nu_r} = \frac{\nu_1 - \nu_r}{\nu_r} \tag{12.83}$$

As an example, if ν_b = 5 Hz and ν_r = 5 MHz then $\nu_b/\nu_r = 1 \times 10^{-6}$ which means that the oscillator frequency ν_1 is either high or low by 1 part in 10^6.

The fractional frequency deviation may be defined as,

$$y_k = \frac{\nu_{bk}}{\nu_r} \tag{12.84}$$

The time deviations can be calculated since,

$$2\pi\nu_1 t = 2\pi\nu_r t + \phi_1(t) \tag{12.85}$$

where $\phi_1(t)$ is the accumulated phase of oscillator 1 with respect to the reference.

Then $$(\nu_1 - \nu_r) \cdot t = \frac{\phi_1(t)}{2\pi} \qquad (12.86)$$

and $$y_1(t) \cdot t = \frac{\phi_1(t)}{2\pi\nu_r} \equiv x_1(t) \qquad (12.87)$$

which is the time deviation of oscillator 1 with respect to the reference.

The average fractional frequency deviation for a finite sample becomes,

$$\bar{y}_1(t_\tau) = \frac{\Delta x_1}{\Delta t} = \frac{x_1(t + \tau) - x_1(t)}{\tau} \qquad (12.88)$$

The disadvantages of the frequency counter are that state-of-the-art oscillators cannot be measured directly, the measurement time is limited and dead-time can present problems.

Tight Phase-Lock Method

The tight phase-lock method of measuring frequency and frequency stability is illustrated in Figure 12.24.

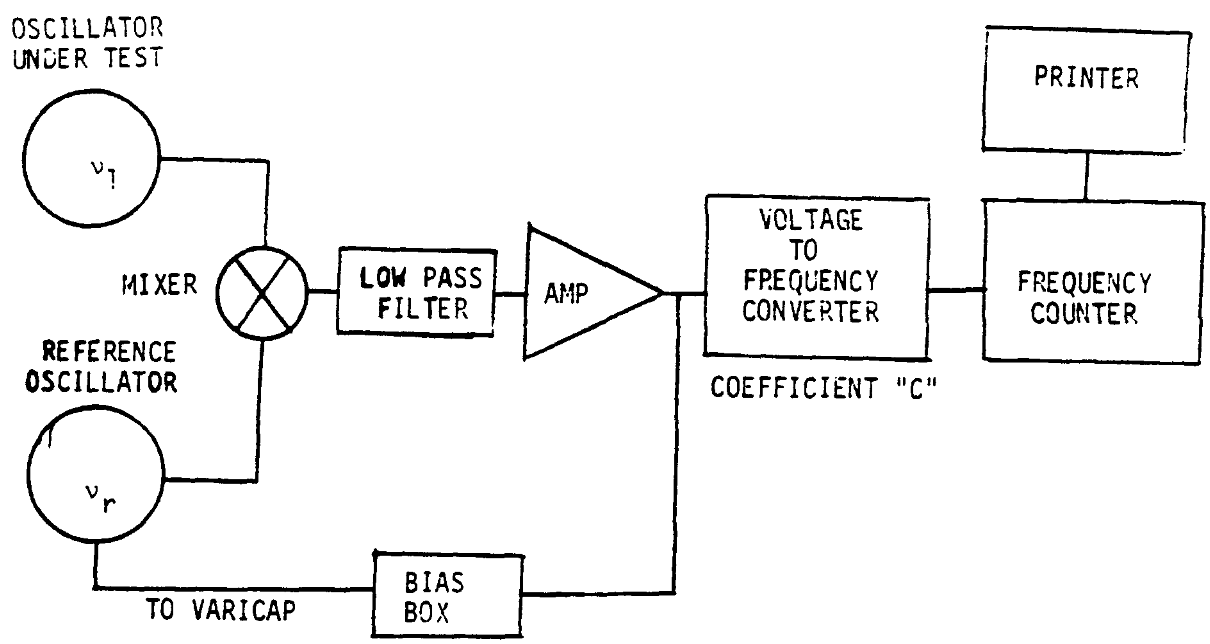

Figure 12.24 Tight Phase-lock Method of Measuring Frequency Stability.

The reference frequency ν_r is forced to phase track the unit under test ν_1. The signals are locked in quadrature and any phase error produces an error signal which is applied in a direction to drive the signal back to the original voltage value. A bias voltage is applied so that fluctuations do not cause the voltage to frequency converter to go through zero volts. The attack time of the phase-lock loop must be much less than the measurement period.

Careful calibration of the varicap (c_v) is required. The Hz/volt calibration of the voltage to frequency converter is set to a desired value.

$$y_k = \text{Printer reading} \times \frac{c_v}{c} \tag{12.89}$$

A measure of stability in the time domain is the infinite time average of the sample variance of two adjacent averages of y(t).

$$\bar{y}_k \equiv \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t)dt = \frac{\phi(t_k + \tau) - \phi(t_k)}{2\pi\nu_0\tau} \tag{12.90}$$

In practice, records are of finite length and the infinite time average implied in the definition is normally unavailable. Therefore, estimates of the two measures must be used.

The measure of frequency stability is one of sample variance where,

$\sigma_y^2(N, T, \tau)$	Sample variance of N averages of y(t), each duration τ, and spaced every T units of time.
$\langle\sigma_y^2(N, T, \tau)\rangle$	Average value of the sample variance $\sigma_y^2(N, T, \tau)$.
$\sigma_y^2(N, T, \tau, f_h)$	Sample variance of N averages of y(t), each of duration τ and repeated every T units of time measured in a post-detection noise bandwidth of f_h.

A practical method of determining spectral density of frequency fluctuations from the variance of the frequency fluctuations, the sampling time, the number of samples taken, and the dependence on system bandwidth was set forth by David Allan.

The definition for the measure of frequency stability is defined by

$$\sigma_y^2(\tau) \equiv \langle\sigma_y^2(N = 2, T = \tau, \tau\rangle \qquad (12.91)$$

where $\sigma_y^2(\tau)$ is the specific Allan variance .

Where N = 2 and the dead time is zero (T = τ)

$$\sigma_y^2(\tau) \equiv \langle\sigma_y^2(N = 2, T = \tau, \tau)\rangle = \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2} \qquad (12.92)$$

$$\approx \frac{1}{2(M - 1)} \sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2$$

where M is the number of data available and M-1 is the number of differences averaged. The bar over the y indicates that y has been averaged over a specified time interval τ. The angular brackets indicate an average over a quantity of time.

The finite average is approximately equal to the infinite average for most kinds of frequency fluctuations encountered [3],[17].

One estimates $\sigma_y^2(\tau)$ from a finite number of values of $\sigma_y^2(2, T = \tau, \tau)$ and averages to obtain an estimate of $\sigma_y^2(\tau)$. One should specify the number of independent samples used for an estimate of $\sigma_y^2(\tau)$. Also, good practice dictates that the system noise bandwidth f_h be specified with any results since the actual shape of the frequency cutoff may be very important.

12.11 FREQUENCY DOMAIN TO TIME DOMAIN TRANSLATION

When dealing with noise processes, a power law in the frequency domain corresponds to a particular power law in the time domain.

The two quantities which specify $\mathcal{L}(f)$ for a particular power law noise process are (1) the slope of the log-log plot for a given range of Fourier frequency (f) and (2) the amplitude. The slope of the plot is denoted as α and the amplitude is denoted by h. the symbol h_α represents the amplitude and noise process. The power spectral density of frequency fluctuations are specified by the addition of all $h_\alpha f$.

Table II can be used for conversion from the frequency domain to the time domain for the Allan variances listed in the two columns on the right.

Translation of data from frequency domain into time domain performance will be illustrated by examples using the following relationships.

$$S_y(f) = \frac{S_{\delta\nu}(f)}{\nu_o^2} \qquad (12.93)$$

$$S_{\delta\phi}(f) = \frac{\nu_o^2}{f^2} \cdot S_y(f) = 2\mathcal{L}(f) \qquad (12.94)$$

$$S_{\delta\nu}(f) = f^2 S_{\delta\phi}(f) \qquad (12.95)$$

Assume that $S_{\delta\phi}(f)$ exhibited approximate $1/f^3$ behavior over the plotted range. Then $\alpha = -1$ and this denotes Flicker FM as indicated in Table I. For the Fourier frequency of 640 Hz, $S_{\delta\nu}(f) = 1.5$ dB and $S_{\delta\phi}(f) = 54.5$ dB. $S_y(f)$ can therefore be calculated by either of the following.

$$S_y(f) = \frac{S_{\delta\nu}(f)}{\nu_o^2} = \frac{10^{1.5/10}}{(9\times10^9)^2} = 1.74 \times 10^{-20}\ \mathrm{Hz}^{-1} \qquad (12.96)$$

where ν_o is 9 GHz and $S_{\delta\nu}(f) = 1.5$dB.

$$S_y(f) = \frac{f^2}{\nu_o^2} \cdot S_{\delta\phi}(f) = \frac{(640)^2}{(9\times10^9)^2} \cdot 10^{-54.5/10} = 1.79 \times 10^{-20}\mathrm{Hz}^{-1} \quad (12.97)$$

Using the value of $S_y(f)$ in Eq. 12, then from column 2 of Table I,

$$S_y(f) = \frac{h_{-1}}{f} \qquad (12.98)$$

$S_y(f) \equiv$ one-sided spectral density of y (dimensions are y^2/f), $0 \leq f \leq f_h$, $f_h \equiv B$, $2\pi f_h \tau >> 1$; $S_y(f \geq f_h) = 0$

General Definition: $$\langle\sigma_y^2(N, T, \tau, f_h)\rangle \equiv \frac{1}{N-1} \sum_{n=1}^{N} (\bar{y}_n - \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k)^2 \quad , \frac{dx}{dt} \equiv y \equiv \frac{\delta\nu}{\nu_0}, r \equiv \frac{T}{\tau}$$

Specific Allan Variance $$\sigma_y^2(\tau) \equiv \langle\sigma_y^2(N = 2, T = \tau, \tau, f_h)\rangle = \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2}$$

	$S_y(f)$	$2\mathcal{L}(f)$ or $S_{\delta\phi}(f)$	$\sigma_y^2(\tau)$	$\langle\sigma_y^2(N, T = \tau, \tau, f_h)\rangle$
Random Walk *FM* $\alpha = -2$	$\frac{h_{-2}}{f^2}$	$\frac{h_{-2}\nu_o^2}{f^4}$	$h_{-2} \cdot \frac{(2\pi)^2\tau}{6}$	$h_{-2} \cdot \frac{(2\pi)^2\tau}{12} \cdot N$
Flicker *FM* $\alpha = -1$	$\frac{h_{-1}}{f}$	$\frac{h_{-1}\nu_o^2}{f^3}$	$h_{-1} \cdot 2 \ln 2$	$h_{-1} \cdot \frac{N \ln N}{N-1}$
White *FM* $\alpha = 0$	h_o	$\frac{h_o\nu_o^2}{f^3}$	$h_o \cdot \frac{1}{2}\tau^{-1}$	$h_o \cdot \frac{1}{2}\tau^{-1}$
Flicker ϕM $\alpha = 1$	$h_1 f$	$\frac{h_1\nu_o^2}{f}$	$h_1 \cdot \frac{1}{\tau^2(2\pi)^2} \cdot [\frac{9}{2} + 3 \ln(2\pi f_h\tau) - \ln 2]$	$h_1 \cdot \frac{2(N+1)}{N\tau^2(2\pi)^2} \cdot [\frac{3}{2} + n(2\pi f_h\tau) - \frac{\ln N}{N^2-1}]$
White ϕM $\alpha = 2$	$h_2 f^2$	$h_2\nu_o^2$	$h_2 \cdot \frac{3f_h}{(2\pi)^2\tau^2}$	$h_2 \cdot \frac{N+1}{N(2\ )^2} \cdot \frac{2f_h}{\tau^2}$

TABLE I. Frequency Domain to Time Doman Stability Measure Conversion Chart.
(Based on Barnes, et al, "Characterization of Frequency Stability," NBS, TN394)

Therefore $h_{-1} = f \cdot S_y(f) = 640 \times 1.74 \times 10^{-20} Hz^{-1}$

$$h_{-1} = 1.11 \times 10^{-17} \quad (12.99)$$

$$\sigma_y^2(\tau) = h_{-1} \cdot 2 \ln 2 = 1.11 \times 10^{-17} \cdot 1.386 = 1.539 \times 10^{-17} \quad (12.100)$$

$$\sigma_y(\tau) = 3.92 \times 10^{-9} \quad (12.101)$$

A plot of the NBS defined spectral density of phase (Script $\mathcal{L}(f)$) is illustrated in Figure 12.25 and shows three power law processes.

Assuming a carrier frequency of 5 MHz the translation to the time domain in regions 1 and 3 can be calculated at Fourier frequencies of 100 Hz and 10 KHz respectively.

For region 1, $\alpha = -1$, since Flicker FM is observed.

$$2\mathcal{L}(f) = \frac{h_\alpha \nu_o^2}{f^3} \quad (12.102)$$

at 100 Hz

$$h_\alpha = h_{-1} = \frac{2\mathcal{L}(f) \cdot f^3}{\nu_o^2} = \frac{2(10^{-12}Hz)(100Hz)^3}{(5x10^{6}Hz)^2} = 8 \times 10^{-2} Hz^{-2} \quad (12.103)$$

The specific Allan Variance is,

$$\sigma_y^2(\tau) = h_{-1} \cdot 2 \ln 2 = (8 \times 10^{-20})(1.386) = 1.11 \times 10^{-19} \quad (12.104)$$

$$\sigma_y(\tau) = 3.33 \times 10^{-10} = \text{Standard deviation} \quad (12.105)$$

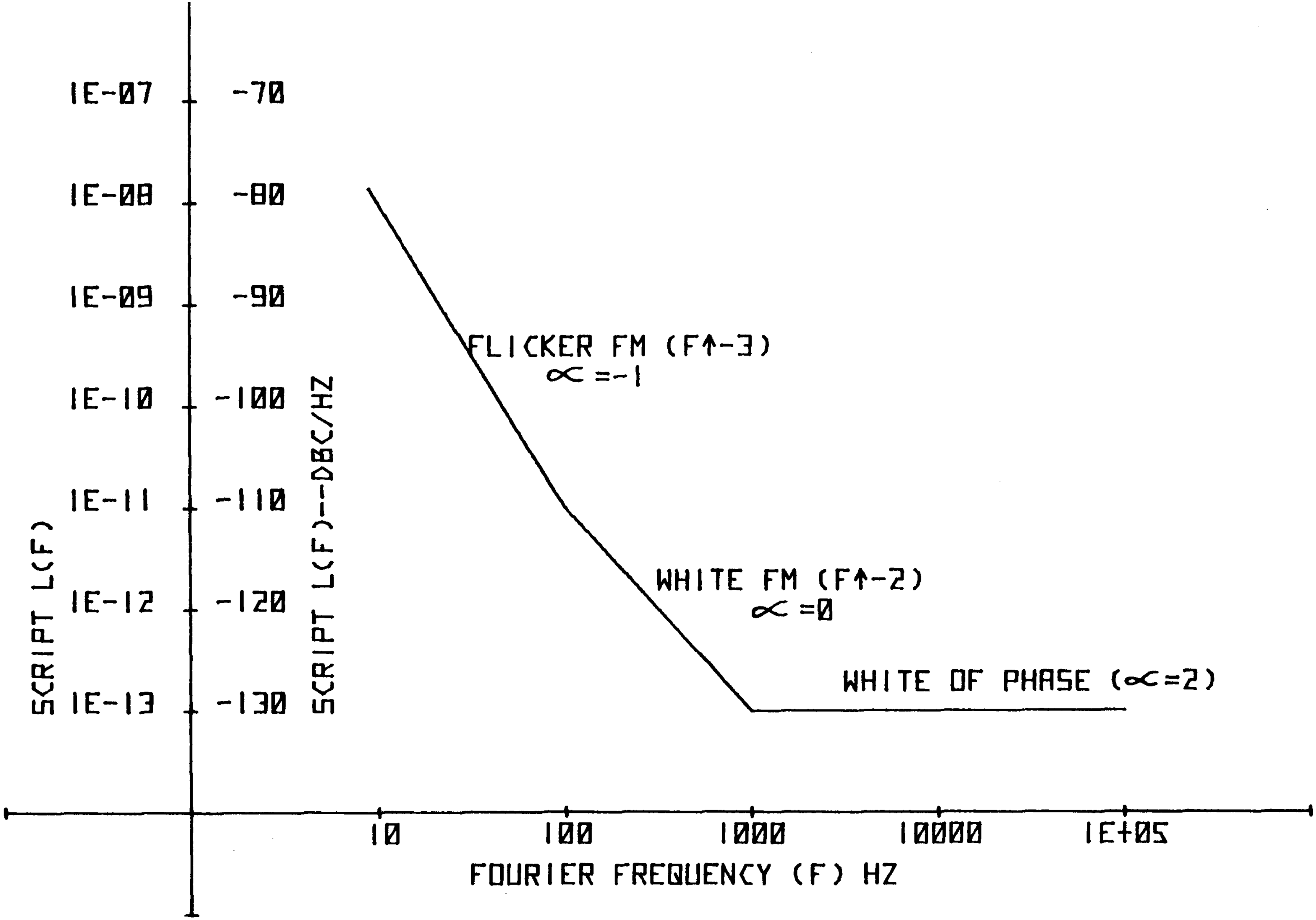

FIGURE 12.25 SPECTRAL DENSITY OF PHASE

For region 3, α = 2, since white phase modulation noise is indicated.

$$h_\alpha = h_2 = \frac{2\mathcal{L}(f)}{\nu_0^2} = \frac{2(10^{-14}Hz)}{(f \times 10^6 Hz)^2} = 8 \times 10^{-28} Hz^{-2} \quad (12.106)$$

$$\sigma_y^2(\tau) = h_2 \cdot \frac{3f_h}{(2\pi)^2\tau^2} = 1.52 \times 10^{-25}\ \tau^{-2} \quad (12.107)$$

for a bandwidth of f_h = 10 KHz,

$$\sigma_y(\tau) = 3.9 \times 10^{-13}\ \tau^{-1} \text{(Standard deviation)} \quad (12.108)$$

12.12 TIME DOMAIN MEASUREMENTS OF PHASE NOISE CLOSE TO THE CARRIER

The Hewlett Packard 5390A Frequency Stability Analyzer can be used to measure close-in phase noise to within 0.01 Hz of the carrier.

The system confiuration is shown in Figure 12.26.

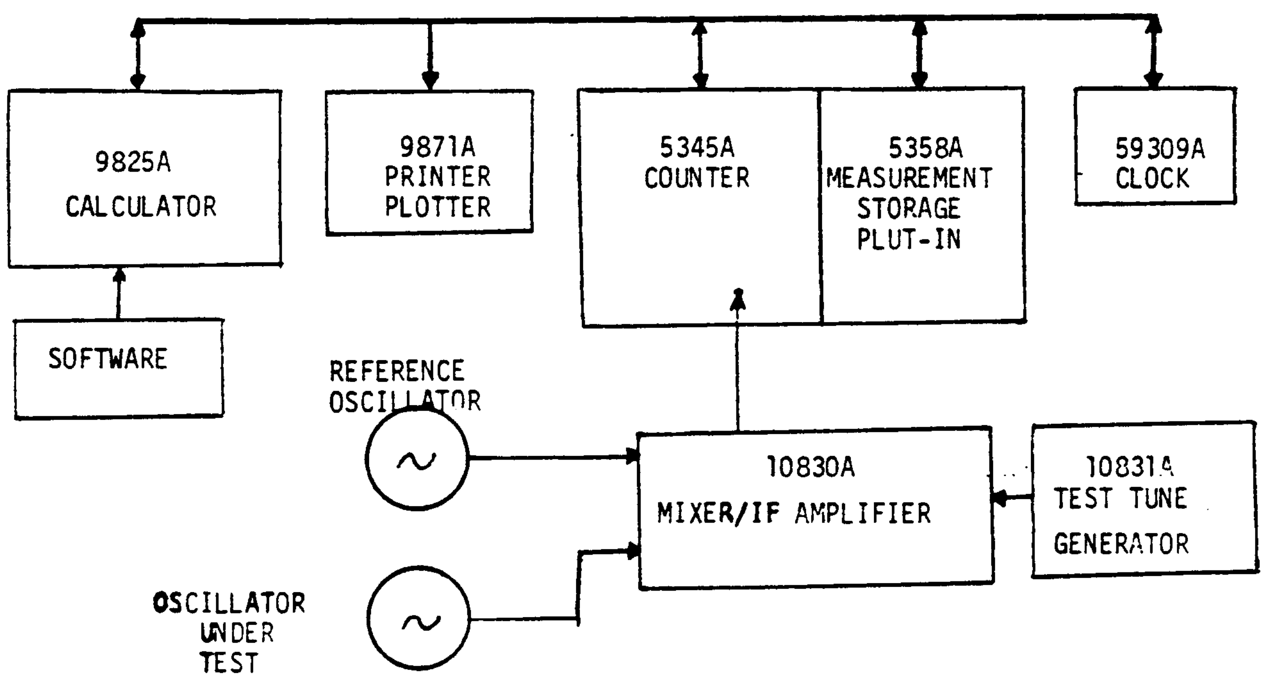

Figure 12.26 The Hewlett-Packard Frequency Stability Analyzer System Configuration.

The oscillator under test is compared with a reference oscillator which is offset in frequency. The outputs of the two oscillators are mixed to produce a low frequency IF beat signal ν_b.

As discussed in a previous section, if the two oscillators are similar it is assumed that the spectral noise density characteristics are the same. This approach always gives a maximum possible error less than 3dB. This can be resolved if three oscillators are compared in all combinations.

The HP10830A Mixer/Amplifier provides the necessary filtering, bandwidth control and low noise, high gain, amplifier to properly condition the signal input to the counter. i.e., to provide reliable detection of the zero crossings of the beat signal ν_b.
The output of the amplifier is essentially a square wave which is used as the input to the HP 5345A reciprocal counter. The input noise bandwidth can be adjusted to be compatible with the signal being measured.

The reciprocal counter makes high resolution (2 n sec) period or time interval measurements. It measures the number of cycles of the input and the elapsed time in 2 n sec steps that it took the n cycles to occur.

The HP 8358A Measurement Storage Plug-in extends several capabilities of the counter. A gate generator in the plug-in generates a measurement time τ_m in the range of 1 to 999 x 10^6 micorseconds. The plug-in also controls the dead time τ_d between measurements. The plug-in stores the measurement data in a buffer memory for output over the interface bus to the calculator for processing.

The calculator is programmed to perform the various measurement operations and to process the data received from the counter and plug-in. The calculator directs the operations of the counter and plug-in via the interface bus.

The printer is used to output the processed results in numeric, or plotted form.

The test tone generator provides a low noise IF frequency to verify system performance by self-checking the system noise floor.

The clock provides time of day for measurement documentation.

The Frequency Stability Analyzer operates as a phase sensitive spectrum analyzer in the frequency domain when used to perform phase noise measurements. The counter takes data in the time domain and the data is transformed into the frequency domain.

The chosen sampling function is the so called Hadamard Variance or 50 percent dead time sampling function suggested by Baugh [23].

The sampling function [24] is illustrated in Figure 12.27. The calculator applies the weighting functions $a_1 = +1$, $a_2 = +1$, $a_3 = 0$, $a_4 = -1$, $a_5 = -1$ and $a_6 = 0$. The sampling function approaches a sine wave so that its transfer function $H_\phi(f)$ will have a single fundamental response in the frequency domain. Referring to Figure 12.27 it is noted that the actual measurement consists of one long reading during the measurement time. This is equivalent to taking two short measurements with zero dead time. For a3 = 0, the actual dead time, the counter outputs data to the plug-in thereby removing these factors from the measurement considerations. The next long measurement is given a weighting factor of -1 by simply subtracting it from the previous reading.

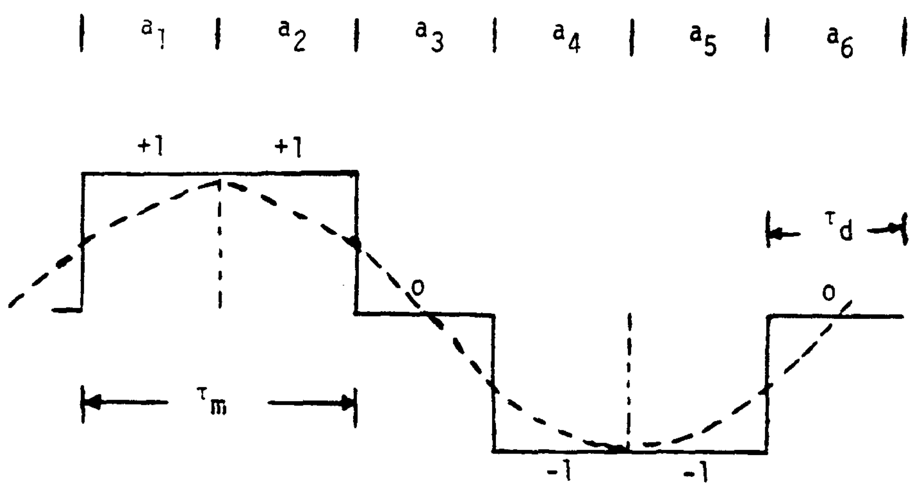

Figure 12.27 Illustration of the Sampling Function.

The counter performs a sequence of N such frequency difference measurements and the calculator stores the sum, M. The sequence is repeated k times.

$$M_1 = (y_{1.1} - y_{1.2}) + (y_{1.3} - y_{1.4}) + \dots + (y_{1.2N-1} - y_{1.2N}) \quad (12.109)$$

$$M_2 = (y_{2.1} - y_{2.2}) + (y_{2.3} - y_{2.4}) + \dots + (y_{2.2N-1} - y_{2.2N}) \quad (12.110)$$

$$M_k = (y_{k.1} - y_{k.2}) + (y_{k.3} - y_{k.4}) + \dots + (y_{k.2N-1} - y_{k.2N}) \quad (12.111)$$

The calculator computes the variance M_k sums

$$\sigma_m^2(\tau_m, \tau_d, N) = \frac{1}{k-1} \left[\sum_{i=1}^{k} M_i^2 - \frac{1}{k} \left(\sum_{i=1}^{k} M_i \right)^2 \right] \quad (12.112)$$

Peregrino and Ricci [22] show that the linear combination of the frequency measurement M(t) can be considered as the output of a filter whose input is $\Omega(t)$ and transfer function $H_\phi(\omega)$ as illustrated in Figure 12.28.

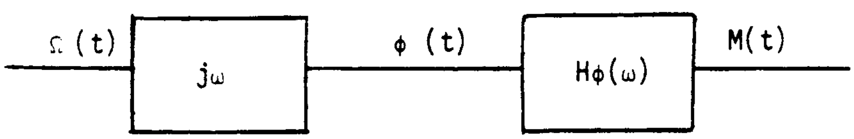

Figure 12.28 Block diagram illustrating input and Tranfer Function Relationship

The relationship between the time domain variance, transfer function and the desired spectral noise density is [22].

$$\sigma_m^2(\tau_m, \tau_d, N) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| H_\phi(\omega) \right|^2 S_{\delta\phi}(\omega)\, d\omega \quad (12.113)$$

using $H_\Omega(\omega)$ instead of $H_\phi(\omega)$,

$$\sigma_m^2(\tau_m, \tau_d, N) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| H_\Omega(\omega) \right|^2 \omega^2 S_{\delta\phi}(\omega)\, d\omega \qquad (12.114)$$

Where an approximation of $H_\phi(\omega)$ is,

$$H_\phi(\omega) \cong \left| \frac{N_{\omega o}}{\pi} \cdot \frac{\text{Sin } r\pi/2}{r\pi/2} \cdot \frac{\text{Sin } N\pi \frac{\omega \pm \omega_o}{\omega_o}}{N\pi \frac{\omega \pm \omega_o}{\omega_o}} \right| \qquad (12.\ 115)$$

where $r = \tau_m/(\tau_m + \tau_d)$ and $\omega_o = \pi/(\tau_m + \tau_d)$.

Since the variance is measured and the transfer function is known, the phase spectral density can be computed as,

$$S_{\delta\phi}(\omega) \cong \frac{\sigma_m^2}{\frac{1}{\pi}\left| H_\phi(\omega_o)\right|^2 \frac{\omega_o}{N}} = \frac{1}{8} \left(\frac{r\frac{\pi}{2}}{\text{Sin } r\,\frac{\pi}{2}} \right)^2 \frac{\sigma_m^{\ 2}}{N f_o^{\ 3}} \qquad (12.116)$$

where $f_o = \omega_o/2\pi$

$$S_{\delta\phi}(f_o) \cong \frac{1}{8} \left[\frac{\pi/3}{\text{Sin } \pi/3} \right]^2 \frac{\sigma_m^{\ 2}}{N f_o^{\ 3}} \qquad (12.117)$$

The filter function $H_\phi(\omega)$ can be made to appear as a very narrow band wave analyzer in the frequency domain. The center frequency of the filter function can be controlled by selecting the measurement time and dead time such that

$$f = \frac{1}{2(\tau_m + \tau_d)} = \frac{1}{6\,\tau_d} \qquad (12.118)$$

The bandwidth B of the measurement can be controlled by varying the number of pairs of N taken.

$$B = \frac{f}{N} \qquad (12.119)$$

The minimum bandwidth depends upon the amount of plug-in memory available and is estimated by [24].

$$B_{min} = \frac{f(18\text{- integer log } (\frac{6}{5}f)}{\text{plug-in memory size}} \qquad (12.120)$$

where the memory size is 2048, 4096, 6144 or 8192.

The maximum offset frequencies are given by

$$f_{max} = \frac{\nu_b}{6}\ ,\ f_{max} = \frac{\nu_b}{2 \times 6}\ ,\ f_{max} = \frac{\nu_b}{3 \times 6}\ \text{etc} \qquad (12.121)$$

since τ_d is limited to an intergral period of the frequency ν_b.

Measurement of the carrier level in the frequency domain is not necessary since the measurements are made on zero crossings in the time domain. The measured value of phase noise is actually a random variable. Therefore, the measurements are statistical in nature and result in an estimate of the mean value of phase noise during a measurement. Larger observation periods will increase the label of confidence in the accuracy.

The sensitivity in dBc/Hz can be expressed in terms of the Fourier frequency f (effect from the carrier) and the desired beat brequency ν_b by the relatinship,

$$\text{Sensitivity in dBc} = -173 + 20 \log \nu_b - 10 \log f \qquad (12.122)$$

A plot of this equation is shown in Figure 12.29.

Figure 12.30 is a plot of phase noise data taken with the HP Frequency Stability Analyzer.

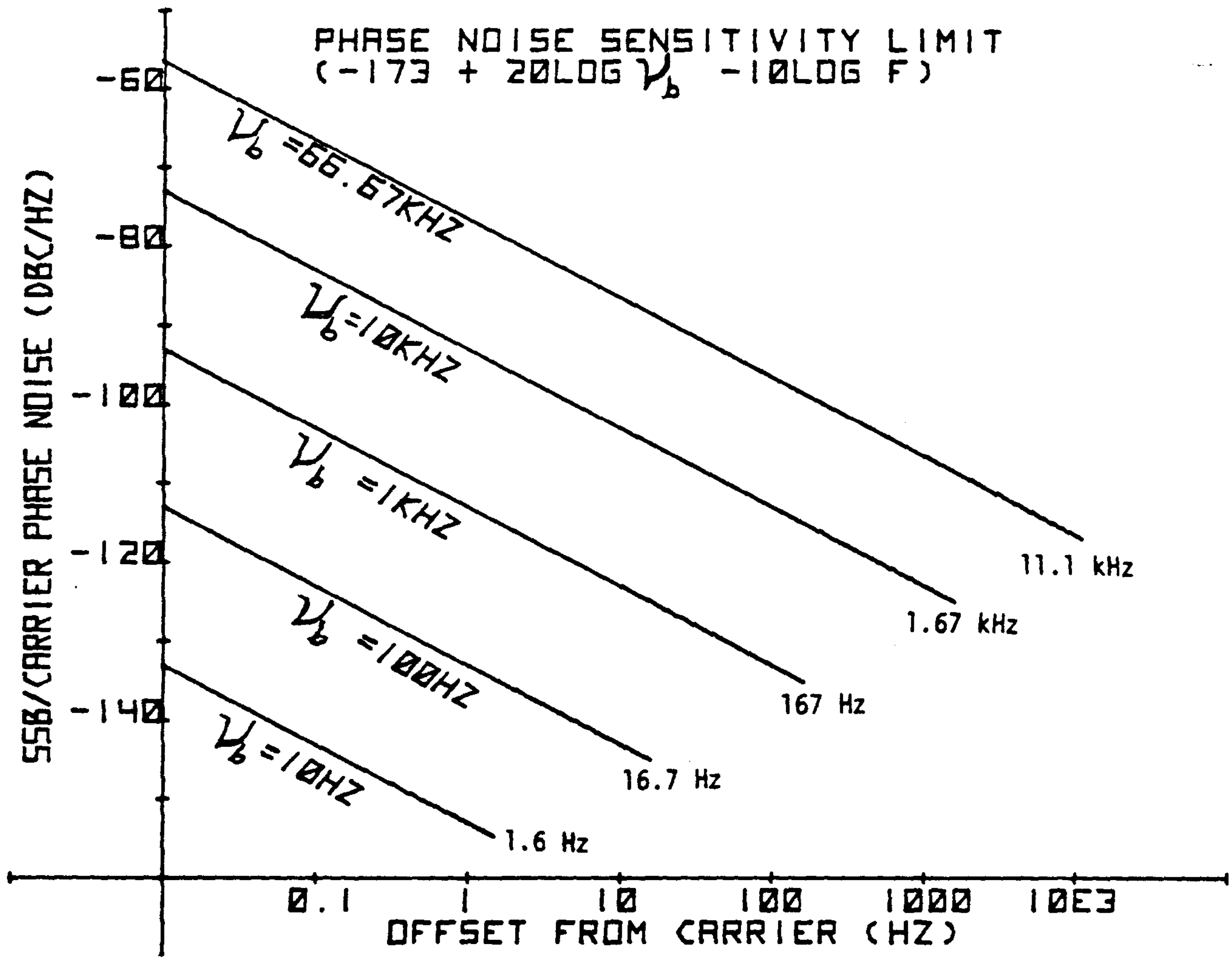

FIGURE 12.29 RELATIONSHIP BETWEEN SENSITIVITY AND FOURIER (OFFSET)

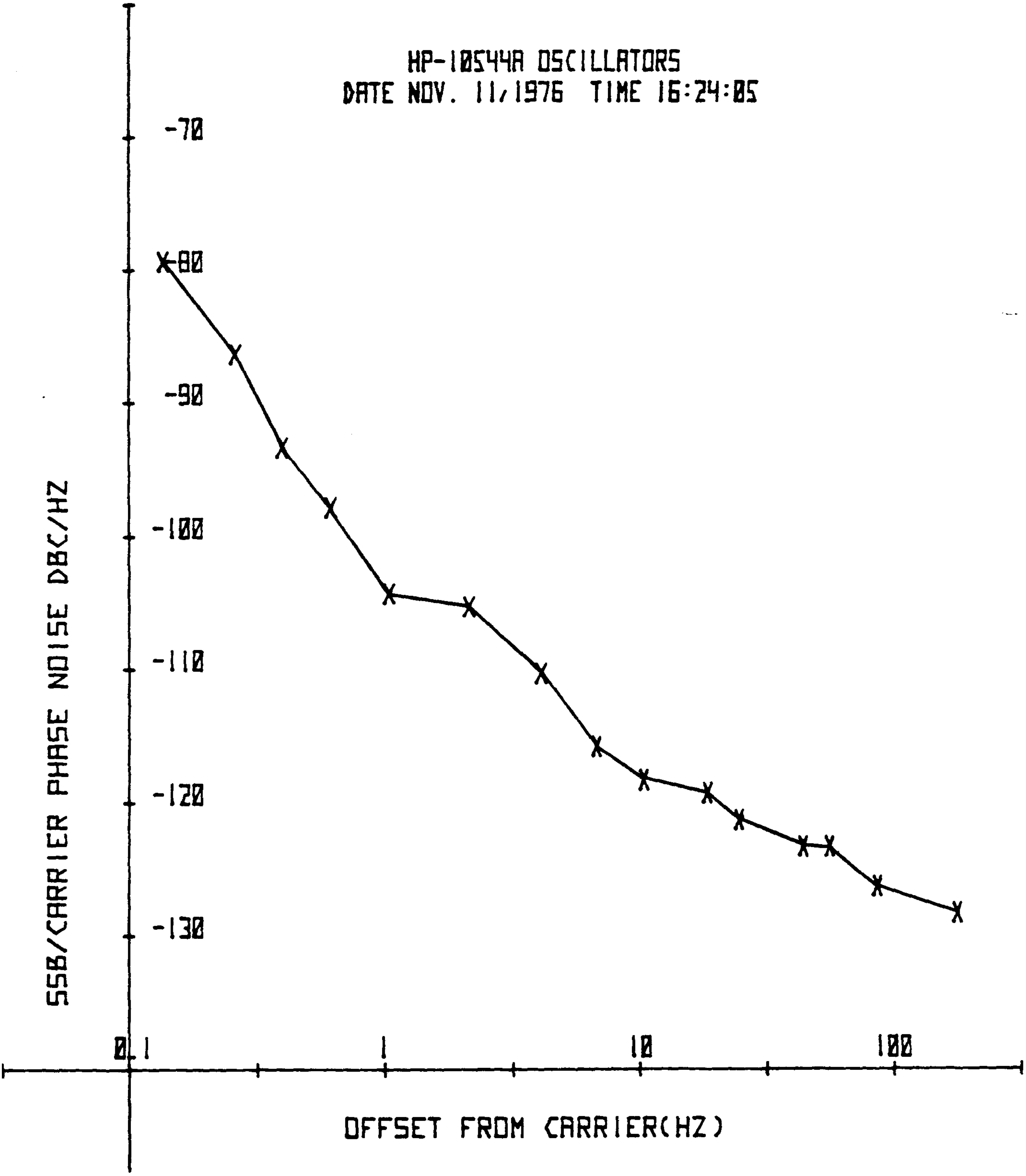

FIGURE 12.30 TIME DOMAIN PLOT-DATA OBTAINED WITH THE HP-5390 SYSTEM. MEASURED AT TRW BY HOLLY COLE OF HEWLETT-PACKARD SANTA CLARA DIVISION.

12.13 PHASE NOISE MEASUREMENT SYSTEM USING A CAVITY DISCRIMINATOR

Measurement System

The basic single oscillator measurement system is shown in Figure 12.31. The basic sections of the system are:

1. Balanced mixer - (amplitude demodulator)
2. Cavity discriminator - (Phase demodulator)
3. Spectrum Analyzer - (baseband noise analyzer)
4. Phase Shifter - (Adjusts the required 90° advance of the carrier with respect to the original carrier and side-bands in the upper arm of the system.
5. Monitoring portion of the system - (Used to set up dc reference levels during calibration and measurement.

Isolators are usually required at the points indicated by I in the figure.

The single oscillator signal is applied to the upper and lower channels (arms) of the system through the directional coupler. Attenuator No. 1 is used to adjust signal levels to the system.

The phase shifter is used to adjust for quadrature of the carrier signal in the lower channel and the noise side-bands in the upper channel.

The circulator and cavity form a low insertion loss discriminator which suppresses the carrier. The ideal carrier suppression filter would provide infinite attenuation of the carrier and zero attenuation of all other frequencies. The practical cavity, used as a filter, has a finite bandwidth that suppresses both carrier and side-bands on both sides of the carrier.

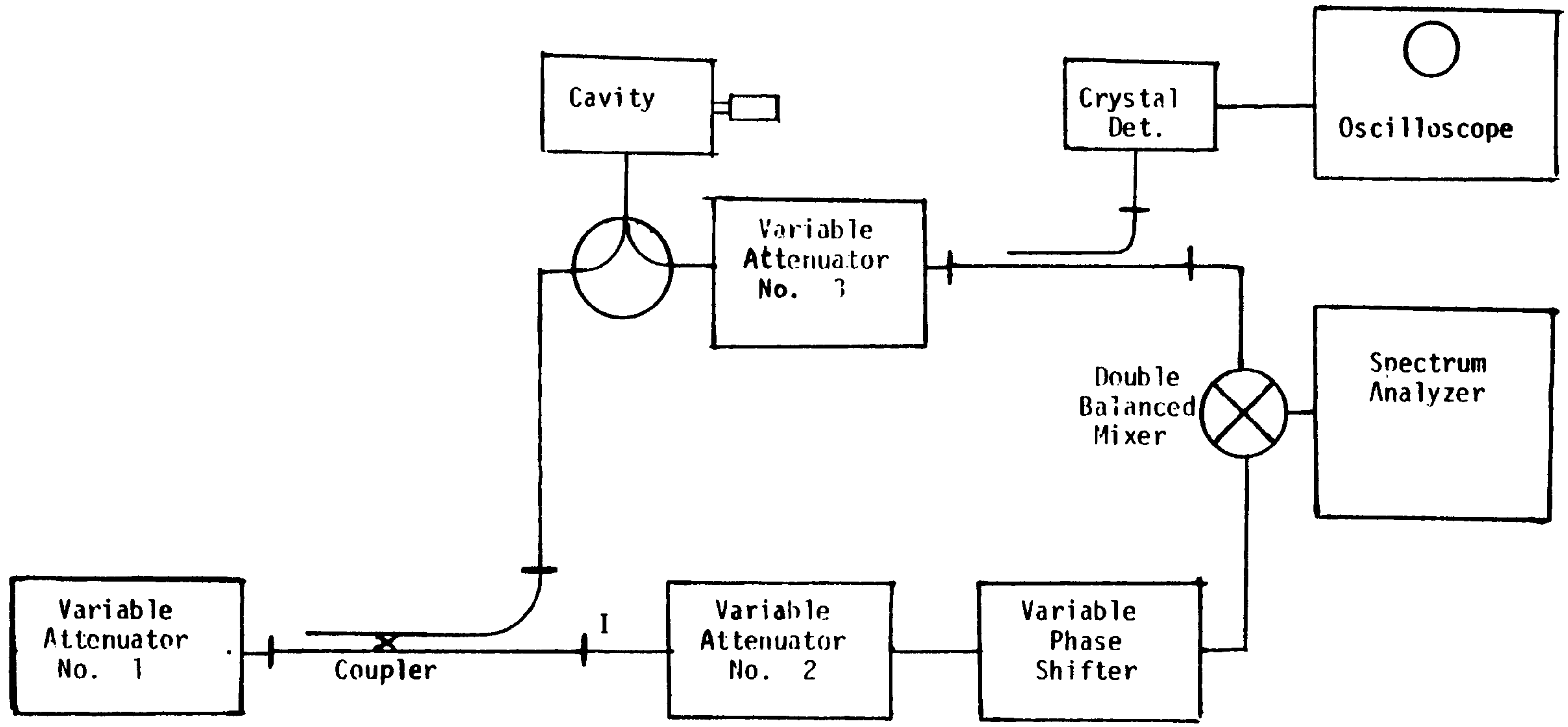

Figure 12.31. Basic Measurement System Using the Cavity Discriminator.

The Q of the cavity determines how much the signals are attenuated. The Q of TE_{011} mode cavities used in this application are usually in the range of 20,000 to 30,000.

The signal enters the circulator and is directed to the cavity which is tuned to the precise frequency of the carrier. The carrier is absorbed by the cavity and, since the cavity acts as a short circuit to all other frequencies, the side-bands are reflected back and directed out to the mixer. The phases of the side-bands are also changed by the cavity (suppression filter). The upper side-bands are advanced in phase and the lower side-bands are retarded by the same mount. Since the carrier is suppressed in the upper arm, the level of the side-bands can be raised without overloading the mixer during measurements. This increase in the relative strength of the noise increases the sensitivity of the measurement system.

The monitoring portion of the system is used to observe cavity tuning to the carrier, tuning for symmetry of the discriminator and in setting up dc reference levels on the oscilloscope.

The mixer is the amplitude demodulator or amplitude detector. It acts as a phase sensitive detector so that when two signals are of the same frequency and in phase quadrature (90°) the output is approximately zero volts dc. FM converted to AM is applied to the spectrum analyzer.

System Calibration

The first objectives in the calibration are to obtain a cavity setting which represents precise cavity resonance to the source under test and to obtain a symmetrical discriminator output.

1. With the source under test applying the input signal, tune the cavity to resonance as indicated on the oscilloscope (dc vertical).

2. Replace the test oscillator with a source that can be swept over the frequency range of resonance.
3. Observe the swept frequency display of the discriminator resonance and adjust the tuner to obtain symmetry if necessary.

Experience informs one that it may be necessary to repeat these steps until the response is symmetrical and tuned to the precise frequency of the source under test.

The system calibration factor CF (the scale factor of the discriminator) is obtained as follows.

1. Connect the oscillator to be measured, adjust the cavity far off resonance, set attenuator No. 3 to zero and obtain a dc output reference level on the oscilloscope.
2. Replace the oscillator under test with a source that can be frequency modulated. Tune this source to the precise frequency of the cavity resonance.
3. Adjust the cavity far off resonance and use atten. No. 1 to obtain the samed dc reference of Step 1.
4. Adjust the cavity to resonance and adjust the phase shifter for zero dc indication on a dc voltmeter connected to the mixer output.
5. Connect the spectrum analyzer to the mixer output and increase the modulation frequency to completely suppress the carrier. This is the first carrier null and represents a modulation index of 2.405.
6. Tune the spectrum analyzer to the modulation frequency and record the rms voltage reading (V_{rms}) of the spectrum analyzer.
7. Calculate the calibration factor as:

$$CF = \frac{\Delta\nu_{rms}}{V_{rms}\ \text{(Spectrum Analyzer)}} \qquad (12.123)$$

where $\Delta\nu_{rms}$ is the rms frequency deviation of the carrier due to the intentional modulation.

$$\Delta\nu_{rms} = \frac{\Delta\nu_{peak}}{\sqrt{2}} = \frac{m f_m}{\sqrt{2}} = \frac{2.405 f_m}{\sqrt{2}} \qquad (12.124)$$

Therefore

$$CF = \frac{2.405 \times f_m}{\sqrt{2}\ \ V_{rms}} \quad \text{Hz/volt} \qquad (12.125)$$

The modulation frequency is selected according to the particular dispersion and bandwidth settings available with the particular spectrum analyzer. Select the f_m which results in a well defined carrier suppression.

Measurement Procedure

After calibration is completed, the source to be tested is attached to the system and the original dc reference level is again obtained with the cavity tuned far off resonance.

The cavity is then adjusted to resonance and the phase shifter is adjusted for zero dc output of the balanced mixer.

The spectrum analyzer is connected to the mixer and the rms voltages v_{1rms} are recorded for the chosen Fourier frequency settings of the spectrum analyzer.

A second set of rms voltage readings (v_{2rms}) an taken at the same Fourier frequencies with the signal attenuated in the signal channel (Atten. No. 3

set to maximum attenuation). This is to record the residual additive noise of the system which is not attributable to the phase noise of the carrier under test. Since noise voltages are uncorrelated, the true noise voltage (v_{rms}) is the square root of the difference of the squares of the two voltages. Therefore, at each Fourier frequency of measurements calculate,

$$v_{rms} = \sqrt{(v_{1rms})^2 - (v_{2rms})^2} \qquad (12.126)$$

The rms fluctuations of carrier frequency are calculated at each Fourier frequency as,

$$\delta\nu_{rms} = v_{rms} \times CF \qquad (12.127)$$

The spectral density of frequency fluctuations is calculated as,

$$S_{\delta\nu}(f) = \frac{(\delta\nu_{rms})^2}{B} \quad Hz^2/Hz \qquad (12.128)$$

Where B is the bandwidth at which the readings were made with the spectrum analyzer.

The spectral density of phase fluctuations is calculated as

$$S_{\delta\phi}(f) = \frac{S_{\delta\nu}(f)}{f^2} \quad radians^2/Hz \qquad (12.129)$$

Measurements performed in any bandwidth can be converted by,

$$\frac{V_1}{V_2} = \sqrt{\frac{B_2}{B_1}} \qquad (12.130)$$

For example, if a signal (V_1) is measured as 60dB below the carrier (V_2) in a 30 Hz bandwidth then

$$\frac{V_1}{V_2} = \sqrt{\frac{1}{30}} = 0.1826 = -14.77\text{dB} \qquad (12.131)$$

and -60 + (-14.77) = -74.77dBc/Hz (12.132)

As an exercise, one should verify the following values from the worksheet in NBS Technical Note 632.

Calibration factor CF = $\frac{8.51 \times 10^3 \text{ Hz}}{0.88 \text{ V}}$ = 9.67×10^3 Hz/V

f(Hz)	f^2 (Hz^2)	B (Hz)	v_{rms} (μV)	$\delta\nu_{rms}$ (Hz)	$(\delta\nu_{rms})^2$ (Hz^2)	$S_{\delta\nu}(f)$ dB ‡	$S_{\delta\phi}(f)$ (dB)*
5×10^3	2.5×10^7 (74 dB)†	100	660	6.38	40.7	0.41 (-3.9 dB)‡	-77.9
640	4.1×10^5 (56 dB)	10	1200	11.6	135,0	1.4	-54.5

* $S_{\delta\phi}(f)$ is tabulated in decibels relative to 1 radian^2 Hz^{-1}

† dB relative to 1 Hz^2

‡ dB relative to 1 Hz^2/Hz

TABLE II. Worksheet from NBS Technical Note 632.

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