Dreaded Interview Questions for Fun

Part I

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Having reached a ***sufficient*** station in life that I have either heard most of the typical interview questions that might come my own direction, or have the wit and nonsense to navigate my way through something that I don't know and talk about the weather or something completely unrelated, this on-going collection of (technical) interview questions is intended just for fun. I encourage any readers to forward me their own ***most favorite*** technical questions at jk@am1.us. Don't get up-tight though, after all, this is just for fun.

Question #1: What is the equivalent of j^j ?

This is very straight-forward to solve if we first write the base j value as $j = e^{j\frac{\pi}{2}}$ based on Euler's formula. Given that substitution, then

$$\left(e^{j\frac{\pi}{2}} \right)^j = e^{-\frac{\pi}{2}} \simeq 0.20789$$

Question #1B: Corollary to #1

What is the equivalent form of $e^{j\frac{\pi}{2}} \ln(j)$?

This is the same as Question #1 except it has been re-cast in a slightly different form. The progression from this form back to Question #1 is given by

$$e^{j\frac{\pi}{2}} \ln(j) = j \ln(j)$$
$$= \ln\left(j^j\right) = \ln\left(e^{-\frac{\pi}{2}} \right) = -\frac{\pi}{2}$$

Question #2: In honor of Al Thiele

Solve (x,y) $\in \Re$ with x≠y $x^y = y^x$

From this starting point, clearly

$$y \ln(x) = x \ln(y)$$

and

$$\frac{y}{\ln(y)} = \frac{x}{\ln(x)}$$

In order to have a solution with x≠y, the quantity $z = x / \ln(x)$ must be multi-valued along the real line. A rough sketch of the solution space can be found by collecting a bit more information.

Specifically, the slope dz/dx must approach infinity as x^+ nears unity because the log function in the denominator will blow up. Similarly, a second derivative computation shows that the slope dz/dx with large x becomes

$$\frac{dz}{dx} = \frac{\ln(x) - 1}{\left|\ln(x)\right|^2} \to \frac{1}{\ln(x)} \; as \; x \to \infty$$

It is also easy to show from this last equation that dz/dx= 0 for x= e.

Moving on to a complete solution to the problem, we can write

$$\frac{\ln(y)}{\ln(x)} = \frac{y}{x} = c$$

where c is an auxiliary variable. Solving this equation leads to

$$x^c = y$$

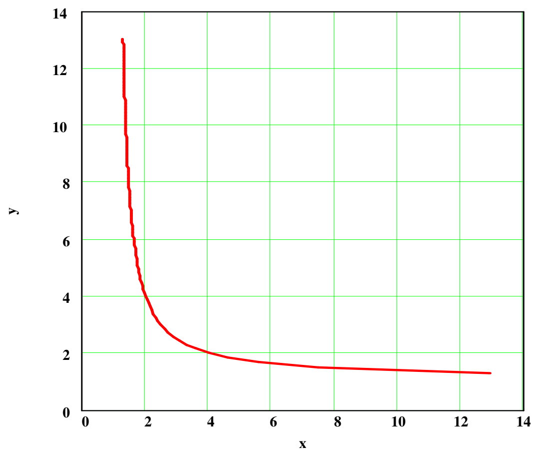
and substitution of this result back into y/x= c leads to

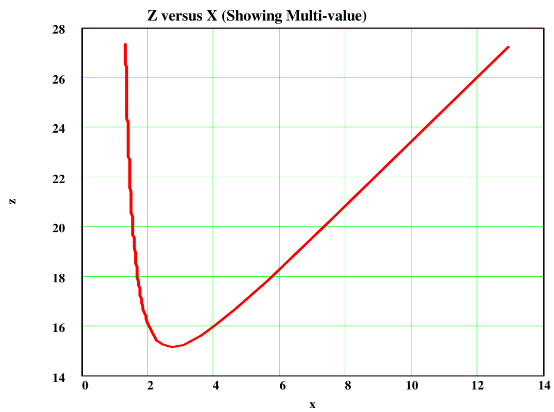
$$x = c^{\frac{1}{c-1}}$$

From this result, it is easy to further conclude that

$$y = c^{\frac{c}{c-1}}$$

The lesson here is to use simple graphical techniques to identify the behavior of a solution when possible. A picture is worth a thousand words.

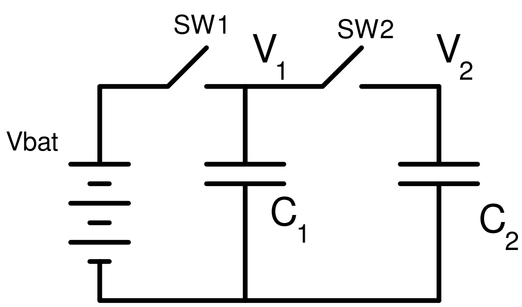




Question #3: Circuits (tricky)

In honor of Fritz Weinert

Consider the simple battery plus capacitor circuit below. Initially, both (ideal) switches are open and both capacitors fully discharged.



Close switch SW1, and capacitor C_1 is charged to voltage V_1= Vbat. Now open switch SW1.

Computing the stored energy in capacitor C_1 results in $E = \frac{1}{2} C_1 Vbat^2$. Now close switch SW2 allowing capacitor C_2 to fully charge. Assuming that $C_2 = C_1$, the voltage will be cut in half on both capacitors resulting in Vbat/2 on each capacitor. The total energy now stored in the two capacitors is

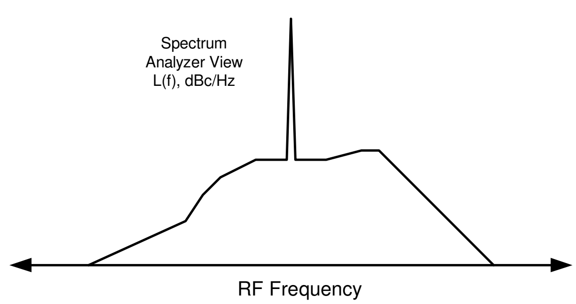
$$E_{tot} = \frac{1}{2} C_1 \left(\frac{Vbat}{2} \right)^2 + \frac{1}{2} C_2 \left(\frac{Vbat}{2} \right)^2$$
$$= C_1 \left(\frac{Vbat}{2} \right)^2 = C_1 \frac{Vbat^2}{4}$$

This all looks reasonable until we realize that the total stored energy originally saved in capacitor C_1 was ***double*** this value. Assuming conservation of energy still holds, where did the other half of the energy disappear to?

The simple fact of the matter is that it is impossible to have two physically separate capacitors and not have some inductance thrown in also! This may seem like a trick question, but many a high-end RF ASIC design tool may have idealized circuit element models involved and the user must be aware of these ideal assumptions. If there were just a bit of inductance included so that the two capacitors could be connected as shown (L), the other half of the energy would reside in a sine wave oscillating at a frequency equal to the series combination of the two capacitors and L. In this (trick) question, the other half of the energy is actually present in an infinite-frequency sine wave as $L \rightarrow 0$.

Question #4: Phase Noise Spectrum

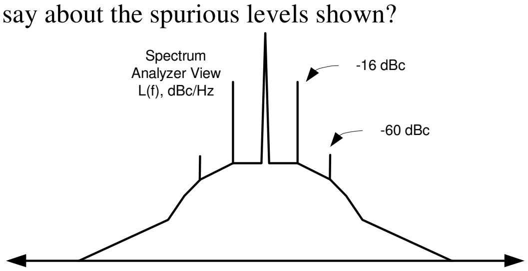
Assume that we were looking at the spectrum of a radio's main local oscillator on a spectrum analyzer and we saw this kind of spectrum. Is there anything wrong with this spectrum, and if so, what?



If this is the output of the radio's local oscillator, it should be constant-envelope in nature. Ideally, the noise pedestal shown above should be completely symmetric about the carrier if only phase noise is present. The asymmetry is a dead give-away that the signal contains both AM and PM noise components. This is true because AM sidebands are in-phase at the same frequency offset from the carrier whereas PM sidebands are precisely opposite phase at the same frequency offset, thereby creating constructive combining on one side of the carrier and destructive combining on the other. Every valid local oscillator spectrum should be symmetric about the carrier center frequency.

Question #5: Another Spectrum Question

Assume as in Question #4 that we are looking at the local oscillator signal from a radio and we see the spectrum below on a spectrum analyzer. The spurious signals on both sides are symmetric as shown. What can we say about the spurious levels shown?



The most obvious question is that if the discrete spurious lines are phase-only, why are the second harmonic spurious terms so much lower in amplitude. If these are truly PM spurs, their levels are dictated by Bessel functions per traditional PM/FM theory, and it is not possible to have the further-out discrete spurious levels this low when the spur levels close-in are this high.

To first-order, if sinusoidal phase modulation with a peak-phase deviation of $\Delta\theta$ radians and frequency f_m is impressed upon the local oscillator somehow, the discrete spurious levels at $\pm f_m$ offset from the carrier are given by

$$L \approx 20Log_{10}\left(\frac{\Delta\theta}{2}\right) dBc$$

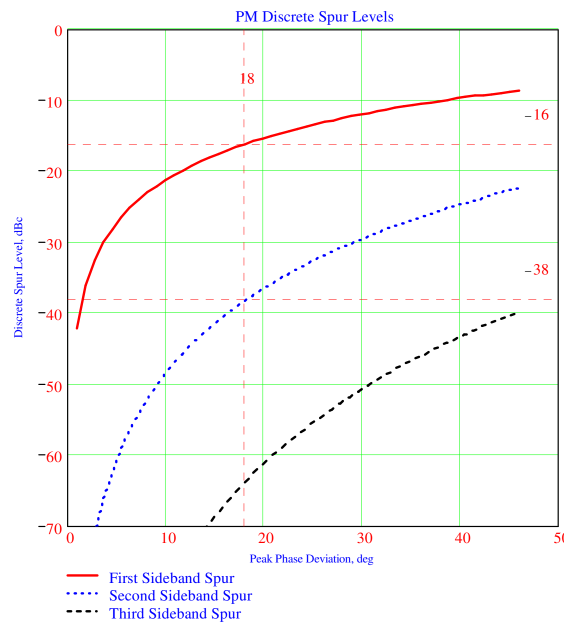
where the quantity $\Delta\theta/2$ is actually an approximation for the J_1 Bessel function amplitude. If the discrete spurs are really due to PM only and the first discrete spur levels have a level of

$$L_1 = 20Log_{10}\left[\left|J_1(\Delta\theta)\right|\right]$$

in general, the n^{th} discrete spur level must have an amplitude of

$$L_n = 20Log_{10}\left[\left|J_n(\Delta\theta)\right|\right]$$

Using these formula, the level of the first few discrete sideband spurs are shown below. As indicated by the dotted lines, if the first discrete spur level is at –18 dBc, the second discrete spur level must be at about –38 dBc if the underlying issue is PM-related spurious.



Coming full-circle then, the discrete spur levels shown in the first figure cannot be PM in nature because they do not obey these relationships. Neither do they appear to be AM and PM related because they are all symmetric about the carrier. We are led to conclude that these are AM-only spurs due to some kind of signal leakage or other undesirable phenomenon happening with the local oscillator.

Question #6: Approximating Time Delays in Systems

To first-order, it is very common to approximate the Laplace transform of a simple time delay as

$$e^{-s\tau} \approx \frac{1}{1+s\tau}$$

but this approximation does not preserve the modulus of the original function. A first-order Pade' approximation can be used to improve upon this approximation as

$$e^{-s\tau} \approx \frac{1-\dfrac{s\tau}{2}}{1+\dfrac{s\tau}{2}}$$

It is tempting to take this approach further and use higher-order Pade' approximates given as

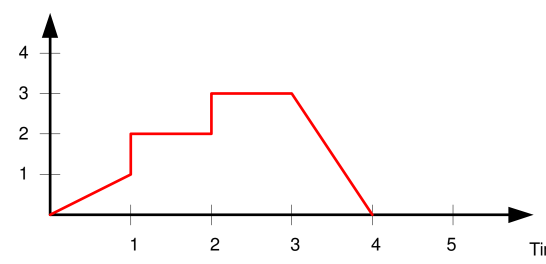
$$e^{-s\tau} \approx \frac{1-s\tau/2+(s\tau)^2/12}{1+s\tau/2+(s\tau)^2/12}\; 2^{nd}\ Order$$

$$\approx \frac{1-s\tau/2+(s\tau)^2/10-(s\tau)^3/120}{1+s\tau/2+(s\tau)^2/10+(s\tau)^3/120}\; 3^{rd}\ Order$$

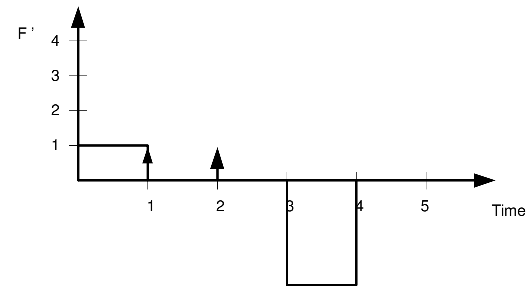
and higher. Are there any potential difficulties if this approximation (with n>2) is used for modeling a system with feedback?

The simple fact of the matter is that the use of denominators with n>1 will in many cases introduce poles into the open-loop gain transfer function that are in the right-half plane thereby introducing potentially serious stability issues. If the system includes feedback, the feedback may effectively move these unwanted right-half plane poles back into the stable left-hand plane region, but there are no guarantees! User beware!

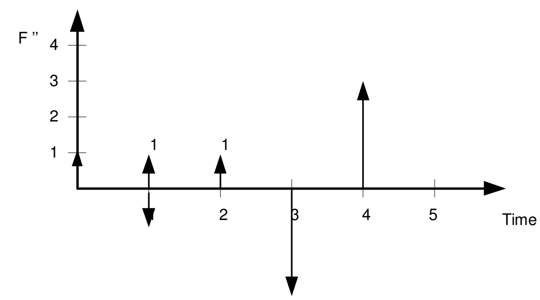
Question #7: Quickly compute the Laplace transform of the time-domain function below.



This is much easier than it might at first seem. First, take enough time derivatives of the function until all that remain are delta-functions and derivatives of delta functions as shown below. After the first time-derivative, we have the function shown below.



After taking the second derivative, we have only delta functions, and derivatives of delta functions as shown below.



The Laplace transform for this impulse string can be written as

$$L\left[\frac{d^2}{dt^2}f(t)\right]=L\left[\delta(t)-\delta(t-1)\right]$$

$$+L\left[\delta'(t)+\delta(t-2)-3\delta(t-3)+3\delta(t-4)\right]$$

$$=1-e^{-s}+se^{-s}+e^{-2s}-3e^{-3s}+3e^{-4s}$$

Knowing that the two derivatives can be undone in the Laplace domain by multiplying this result by s^{-2}, the final result is simply given by

$$L(s)=\frac{1-e^{-s}+se^{-2}+e^{-2s}-3e^{-3s}+3e^{-4s}}{s^2}$$

Question #8: Numerical Computing

Why might we have problems computing the following equation recursively on a computer or digital signal processor

$$x_{n+1}=x_{n-1}-x_n$$

starting with x_0= 1 and x_1= 0.61803398? The first few computed values for n are shown below:

n	x_n
0	1
1	0.61803
2	0.38197
3	0.23607
4	0.14590
5	0.09017
6	0.05573
7	0.03444
8	0.02129
9	0.01316

A little investigation into this recursion shows that the finite difference equation involved has a characteristic equation of

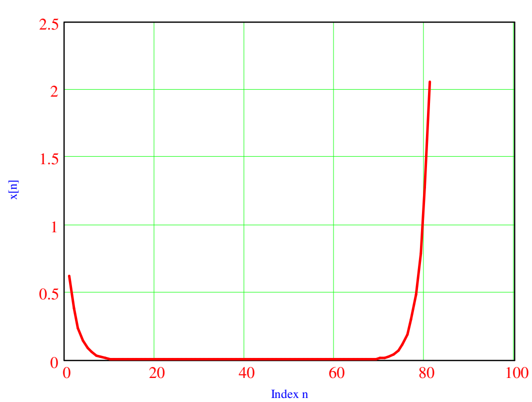
$$z^2 - z - 1$$

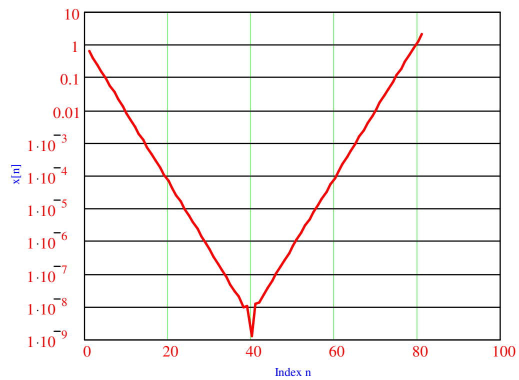
The characteristic equation roots are

$$\left(1 \pm \sqrt{5}\right)/\ 2 = [-0.61803398, 1.61803398]$$

one of which is inside the unit-circle and one which is clearly outside. Starting with x_1= 0.61803398 (which is the stable root) hides the fact that due to finite numerical precision in the recursion computation, round-off errors are effectively increased by the factor 1.618… for every iteration. As a result, the x-values appear to properly decay with n, but if larger values of n are computed, the recursion blows up. This phenomenon is shown below graphically.

If a slightly different starting value has been used, the unstable root would have made its presence known much earlier.





The moral of the story is to always be careful about numerical instabilities or round-off problems that finite precision may be introducing into a problem. Many a digital signal processing algorithm has fallen prey to precision problems. Equally true is that some algorithms will only work properly when noise is present.

Question #9: Stochastic Processes

What kind of stochastic processes satisfy the Weiner-Khinchine Theorem?

Only wide-sense stationary processes satisfy this theorem which states that the power-spectral density is the Fourier transform of the process's auto-correlation function.

Question #10: Matrices

If a square matrix is symmetric, what must be true of its eigenvalues? If on the other hand, one of the eigenvalues is identically equal to zero, what is also true of the matrix?

If the matrix is symmetric, the matrix is called a Toeplitz matrix and all of the eigenvalues are real. If on the other hand one of the eigenvalues is zero, the matrix is singular and not of full rank.

Question 11: Numerical Simulation

What is the bilinear transformation and how is it used?

Many linear systems can be represented in terms of Laplace transforms using the differential operator s. A convenient way to convert the Laplace transform into a finite difference equation (in time) is to use the first-order Pade' approximation which is given by

$$e^{-sT} \approx \frac{1-\frac{sT}{2}}{1+\frac{sT}{2}}$$

After some re-arrangement of terms and letting z= exp(sT), this can be transformed into

$$s \approx \frac{2}{T}\frac{z-1}{z+1}$$

and then substituted back into the Laplace transform for each occurrence of s. Viewing z^{-1} as a simple time-delay operator in a sampled control system, the difference equation(s) can be easily ascertained.

Question #12: Numerical Simulation

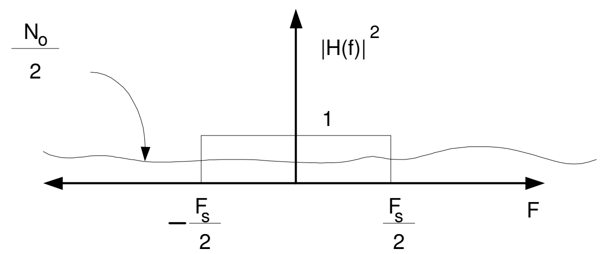
Assume that we need to simulate an AWGN noise spectrum with Gaussian statistics for a simulation that we wish to run and that the sampling rate for the system is assumed to be F_s Hz. How do we proceed?

The most illuminating way that I have seen to look at this problem is to view it as a filtering problem coupled with the Nyquist theorem as follows. Assume that the AWGN continuous-time noise power spectral density is given by $N_o/2$ (a two-sided spectral density). With a sampling rate of F_s, we know that all frequencies in the system must be limited to $F_s/2$ per the Nyquist criterion. To make this true for the noise also, the AWGN noise should be filtered by a brick-wall lowpass filter having a corner frequency of $F_s/2$ as shown below.

The power spectral density out of the ideal lowpass filter is given by

$$S(f) = \frac{N_o}{2} rect\left(\frac{2f}{F_s}\right)$$

where rect(x)= 1 for |x|≤ 1.



Assuming that the process is wide-sense stationary, the Weiner-Khinchine theorem can be used to compute the autocorrelation function as the Fourier transform of S(f) as

$$R(\tau) = \int_{-\infty}^{+\infty} S(f) e^{j2\pi f\tau} df = \frac{N_o}{2}\int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} e^{j2\pi f\tau} df$$

$$= \frac{N_o}{2}\frac{\sin(\pi F_s \tau)}{\pi\tau} = \left(\frac{N_o F_s}{2}\right)\frac{\sin(\pi F_s \tau)}{\pi F_s \tau}$$

It is worthwhile to point out that R(0)= $N_oF_s/2$ which is the variance of the noise samples at the output of the ideal lowpass filter and R(τ)= 0 for all other τ-values corresponding to multiples of T= $(2F_s)^{-1}$. F_s is the Nyquist sampling rate in this case. This latter statement implies that all noise samples at the ideal lowpass filter output will be uncorrelated as desired. In order to simulate the continuous-time system having a flat noise power spectral density level of $N_o/2$ (2-sided as shown above), we only need to use a Gaussian random noise source to create samples having a variance of

$$\sigma^2 = \frac{N_o}{2F_s}.$$

Question #13: Signal Processing

When do Toeplitz matrices appear in signal processing?

One common appearance of Toeplitz matrices is in connection with wide-sense stationary random processes which have single-parameter autocorrelation functions such that $R(\tau) = R(-\tau)$. Such matrices have symmetry about the main diagonal with values along each diagonal equal. Furthermore, the eigenvalues of the matrix are all real.

Question #14: DPSK

Assuming that the local oscillator phase noise spectrum is like that shown in Question #5, estimate the performance loss due to the phase noise as a function of SNR.

The key point here is that differential PSK is being used. We therefore care about the amount of phase accumulated over a symbol interval due to the phase noise. Assuming a wide-sense stationary phase noise spectrum, we can write

$$\sigma_\theta^2 = E\left\{\left[\theta(t+T)-\theta(t)\right]^2\right\} = 2\left[R_\theta(0)-R_\theta(T)\right]$$

From the Wiener-Khinchine theorem, we also know that

$$R_\theta(\tau) = \int_{-\infty}^{+\infty} S_\theta(f) e^{j2\pi f\tau} df$$

This fact can be substituted back to yield

$$\sigma_\theta^2 = 8\int_0^{+\infty} S_\theta(f)\sin^2(\pi f T) df$$

by taking advantage of the phase noise spectral symmetries involved where T is the modulation symbol duration in time.

The phase noise is a multiplicative-type of noise that results in additional noise proportional to the desired signal. As such, the effective SNR including phase noise is given approximately by

$$SNR_{eff} = \left[\frac{1}{SNR} + \sigma_\theta^2\right]^{-1}$$

These last two equations provide an avenue to quickly estimate the impact of local oscillator phase noise on the DPSK system performance.

Question #15: BPSK

Provide an argument that shows that local oscillator phase noise performance should not be very critical for a BPSK system.

One simple argument that can be made is that the modulation amounts to multiplying a sine wave by effectively ±1 or equivalently on-off keying applied to sine wave carriers that are 180 degrees out of phase.

More rigorously however, we know that the BER for uncoded BPSK using coherent demodulation is given by

$$BER = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\cos(\theta)\right)$$

Since the Taylor series expansion for cos(θ) goes as

$$\cos(\theta) \approx 1 - \frac{\theta^2}{2} + \ldots$$

the cosine term is nearly unity unless the phase error gets quite large. Equivalently, the phase noise performance of the local oscillator would have to be quite poor in order to degrade BPSK coherent demodulation performance.

Question #16: Integration by Parts

Quickly derive the integration by parts formula.

Assuming that we have the product of two variables u and v, the total differential is given by

$$d(uv) = u\,dv + v\,du$$

Taking the first term on the right-hand side to the other side of the equation and integrating, we quickly get

$$\int v\,du = uv - \int u\,dv$$

Question #17: State Variables

Given the Laplace transform system transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

is this a lowpass or highpass transfer function and why? Under what conditions is the system stable? Set up a set of state-variable differential equations to solve the output versus input time-domain solution.

The system is unquestionably lowpass because it has unity gain at zero frequency, tending to zero gain at infinite frequency.

The system is stable so long as the damping factor ζ is greater than zero.

In order to set up the set of system equations, simply start by writing H(s)= $V_o(s)/V_i(s)$ and equate this to the transfer function. Then cross-multiply each side of the equation to eliminate all of the denominator terms and finally divide both sides of the equation through by s. The net result is

$$\left(s + 2\omega_n \varsigma + \frac{\omega_n^2}{s} \right) V_o = \frac{\omega_n^2}{s} V_i$$

Recalling that (1/s) is equivalent to integration in the time domain, after collecting terms, we can write

$$sV_o = \frac{\omega_n^2}{s} V_i - 2\omega_n \varsigma V_o - \frac{\omega_n^2}{s} V_o$$

Now let

$$U_1(t) = \int V_o(t) dt$$

$$U_2(t) = \int V_i(t) dt$$

The system of equations can be written as

$$\frac{dU_1}{dt} = V_o(t)$$

$$\frac{dU_2}{dt} = V_i(t)$$

$$\frac{dV_o}{dt} = \omega_n^2 U_2(t) - 2\omega_n \varsigma V_o(t) - \omega_n^2 U_1(t)$$

Question #18: Lagrange Multipliers

Use the method of Lagrange multipliers to show that the area of a fixed-perimeter (P_o) rectangle with dimensions L by W is maximized by a square.

$$A = LW$$

$$P = 2(L + W)$$

$$\Lambda = A + \lambda (P - P_0)$$

$$\frac{d\Lambda}{dL} = W + 2\lambda = 0$$

$$\frac{d\Lambda}{dW} = L + 2\lambda = 0$$

From this, W=L=-2λ so should be a square.

Question #19: Noise Bandwidth

Compute the equivalent noise bandwidth for a single-pole lowpass filter, equivalently a first-order Butterworth filter.

The power transfer function for a single-pole lowpass filter is given by

$$\left| H(f) \right|^2 = \frac{1}{1 + \left(\frac{f}{F_c} \right)^2}$$

where F_c is the –3 dB corner frequency. Assuming that the input noise spectrum is white, the equivalent noise bandwidth is computed simply as

$$BW_{noise} = \int_{-\infty}^{+\infty} \frac{df}{1 + \left(\frac{f}{F_c} \right)^2}$$

$$= 2F_c \int_0^{+\infty} \frac{du}{1 + u^2} = 2F_c \tan^{-1}(u) \Big|_{u=0}^{u=+\infty} = \pi F_c$$

Question #20: Math

What are the first few terms of

$$(a + b)^n \approx ??$$

This is the binomial series and the first few terms are therefore

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!} a^{n-2} b^2 + \ldots$$

Question #21: Math

What are the first few terms of

$$\frac{1}{\sqrt{1 + x}} \approx ??$$

Making use of the Taylor series expansion, the first few terms of this are

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{4*2!} - \frac{15x^3}{8*3!} + ...$$

Question #22: Antennas

What are some of the major assumptions behind the antenna gain formula that is commonly used:

$$G_{Ant} = \frac{4\pi A}{\lambda^2}$$

The major assumptions are that (i) the aperture represented by area A is uniformly illuminated and (ii) this is the maximum gain taken to be normal to the aperture A. In general, it is very difficult to realize antenna gains greater than the value predicted by this formula in the case of aperture-type antennas.

Question #23: Communications

The bit error rate formula for coherent BPSK is

$$BER = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$

Define the erfc(x) function and provide at least one approximation for it.

Erfc(x) is the complementary error function which has a precise value given by

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

$$\approx \frac{e^{-x^2}}{x\sqrt{\pi}} \left(1 - \frac{1}{2x^2} + \frac{1\bullet 3}{2^2 x^4} - \frac{1\bullet 3\bullet 5}{2^3 x^6} + ...\right)$$

Wozencraft & Jacobs show that

$$Q(y) = \frac{1}{2} erfc\left(\frac{y}{\sqrt{2}}\right)$$

and

$$\frac{e^{-\frac{y^2}{2}}}{y\sqrt{2\pi}}\left(1 - \frac{1}{y^2}\right) \le Q(y) \le \frac{e^{-\frac{y^2}{2}}}{y\sqrt{2\pi}}$$

It turns out that the Chernoff bound for the tail probability is given by

$$\Pr ob[x \ge t] \le e^{-\frac{t^2}{2\sigma^2}}$$

in the case of a mean-zero Gaussian random variable with variance σ^2.

Question #24: Random Processes

Given a stochastic random variable x and its probability density function p(x), define its characteristic function.

The characteristic function is simply the Fourier transform of p(x).

Question #25: Math

$$\sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} = 1$$

$$\sum_{n=1}^{N} (2n-1) = N^2$$

Question #26: Gaussian Random Variables

Assuming that x is a mean-zero Gaussian random variable with variance σ^2, what are its first, second, third, and fourth-order moments?

The first moment is the mean and is simply zero. The second moment is the variance and is likewise already given. The third moment is zero because the distribution is symmetric. By using characteristic functions, it is possible to show that the fourth-order moment is given by $3\sigma^4$.

Question #27: Leeson's Model

What is Leeson's model mathematically, and what underlying assumptions apply?

Leeson's model is used to estimate the phase noise spectrum from a free-running oscillator. The single-sideband phase noise power spectral density is given by

$$L(f)=\frac{FkT}{2P_o}\left[1+\left(\frac{F_o}{2Q_L f}\right)^2\right]$$

where

F=	Noise factor
k=	Boltzmann constant
T=	290 Kelvin
P_o=	Power extracted <u>from resonator</u>, W
F_o=	Oscillator center frequency, Hz
Q_L=	Resonator loaded Q
f=	Frequency offset from carrier, Hz

Question #28: Laplace Transforms

Final value theorem for Laplace transforms?

$$f(t)\Big|_{t\to\infty}=\lim_{s\leftarrow 0}\left[sF(s)\right]$$

Question #29: z-Transforms

Final value theorem for z-transforms?

$$\lim_{t\to\infty} f(t)=\lim_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$$

Question #30: Poisson Sum Formula

What is the Poisson Sum Formula?

$$\sum_n Tf(nT)e^{-j\omega nT}=\sum_m F\left(\omega+\frac{m2\pi}{T}\right)$$

Question #31: Filters

Under what circumstances would it be advisable to use a (i) Butterworth filter, (ii) Chebyshev filter, (iii) Gaussian filter, (iv) Bessel filter, or (v) elliptic filter?

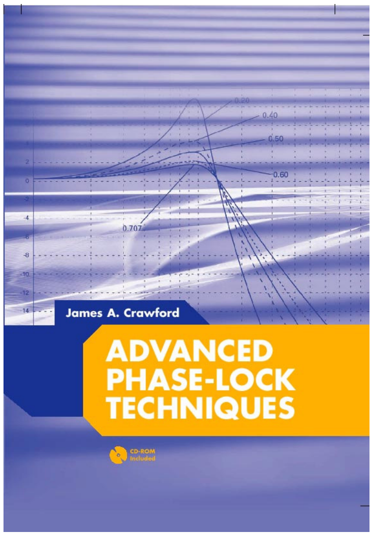
Butterworth filters are maximally-flat at f=0, rolling off to –3 dB at the corner frequency. The poles are equally distributed around a circle in the s-plane.

Chebyshev filters are equi-ripple in the passband. Although the amplitude remains close to unity across the passband, the group-delay characteristics are more extreme than the Butterworth filter. This is particularly true for filter orders greater than 3. Chebyshev poles lie on an ellipse in the s-plane and as such the quality factor of the poles is higher than those of the Butterworth family.

Gaussian filters are also called transitional filters. There are Gaussian "to 6 dB" filters and Gaussian "to 12 dB" filters, etc. A completely Gaussian shaped filter response is physically not realizable with passive capacitors and inductors because it does not satisfy the Paley-Wiener condition. These filters are attractive because of their very benign nonlinear group delay characteristics as compared to Butterworth or Chebyshev filters, historically finding widespread use in radar systems where flat group delay is important.

Bessel filters have a maximally-flat group delay characteristic. Their attenuation characteristics are not very selective, certainly compared to Butterworth or Chebyshev filters, but their group delay characteristics are excellent.

The elliptic filter is the only filter type listed that has both poles and zeros in its transfer function. This filter type delivers equi-ripple passband and stopband attenuation performance, and exhibits the fastest possible transition characteristic between passband and stopband of the filters listed. In exchange for the exceptionally fast transition region performance region possible, group-delay characteristics are quite nonlinear near the corner frequency.



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